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Paul K. Chang

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The human nervous system, including the brain, is the most efficient and versatile data processing system known; and data processing is an essential part of communications. The artificial neuron provides a new approach to investigating and understanding basic nerve network functions. It is a fresh example of how Bell Telephone Laboratories constantly explores new frontiers to improve America's communications system, now and in the years ahead.

Network of neurons is assembled by L. D. Harmon of Bell Laboratories, the initiator of this new research. Many assemblies are possible.

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Vol. 270 NOVEMBER, 1960 No. 5

# PROCESSES INVOLVED IN ELECTROMAGNETIC ACCELERATION OF PARTICLES TO COSMIC-RAY ENERGIES *

BY

W. F. G. SWANN [1]

ABSTRACT

The paper extends ideas, put forward by the writer some twenty-five years ago, according to which charged particles could be accelerated to cosmic-ray energies through electromagnetic induction resulting from magnetic fields like those encountered in sunspots.

The paper confines itself, for simplicity, to cases of axial symmetry, and in particular to cases where $U_\theta$ is of the form $U_\theta = 10^{12}(r_0/r)^n F(t)$, where $r$ is the distance from the axis of symmetry, $U_\theta$ is the vector potential, which lies in planes perpendicular to that axis, and has no radial or $z$ component. $F(t)$ is a function of time which rises from zero at $t = 0$ to a peak value of unity, and then decays to zero. The above expression for $U_\theta$ corresponds to a maximum average magnetic flux of 2000 gauss within the radius $r_0$ which is taken as $10^9$ cm. We consider protons which start from rest at $t = 0$.

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Presented at the Cosmic Ray Conference held in Moscow, July 6–11, 1959.

The present paper is an extended version of the paper as published in the Proceedings of the aforesaid conference. Opportunity is here taken to correct certain minor errors in the proceedings as printed. These errors are purely typographical and do not affect any of the conclusions. They are as follows:

In Eq. 4, the square bracket should have an exponent 1/2; the 2's in the square bracket should be replaced by $c^2$, and the dot should be deleted from $\dot{r}$ in the last term of Eq. 4.

Equation 6 should have a factor $W$ on the left-hand side, and a minus sign on the right-hand side.

$\partial U_\theta/\partial r$ in Eq. 8 should read $\partial U_\theta^2/\partial r$.

I may remark that these typographical errors are the fault of the present writer, and not of the Editors of the Proceedings of the symposium.

[1] Director Emeritus, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa., and a Senior Staff Advisor for The Franklin Institute Laboratories for Research and Development.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

When $\partial U_\theta/\partial r$ is zero, corresponding to $n = 0$, the particles describe circles. This is a good condition for the acquirement of high energies and yields an energy of $3 \times 10^{14}$ ev by the time $F(t)$ has attained the peak value. However, the particles remain in their circular orbits during the deceleration period following the attainment of the peak value unless they are scattered out of them by some scattering mechanism; and they lose all their energy by the time $F(t)$ has diminished to zero.

If $\partial U_\theta/\partial r$ is negative (that is, if $n$ is positive) the particles spiral outwards, while if it is positive (that is, if $n$ is negative) they spiral inwards. However, if $n$ is appreciably different from zero in the positive sense, the particles spiral outwards so rapidly to the realms where the acceleration process is weak that the total energy acquired in spiralling out to infinity is small. However, in order to realize an essentially circular orbit, it is not necessary to have a field for which $\partial U_\theta/\partial r$ is zero at all points. In fact, if the curve of $U_\theta$ plotted against $r$ shows, at some value of $r$, a sharp descent followed by a sharp rise, and by a subsequent slow descent, we have a sort of trough in the $U_\theta$ *versus* $r$ curve. In this trough there is a place where $\partial U_\theta/\partial r$ is zero, and particles can describe essentially stable circular orbits in this trough and can become accelerated to high energies. If during the period of the rise of $F(t)$, which in practice is of the order $10^4$ seconds, the trough disappears, the particles will proceed to spiral out to infinity, but without parting with the energy they have attained while in the trough.

Twenty-five years ago, the writer suggested a method by which cosmic-ray energies could be realized by electromagnetic induction in stellar phenomena associated with changing magnetic fields, the principle involved being that which has come to be known as betatron

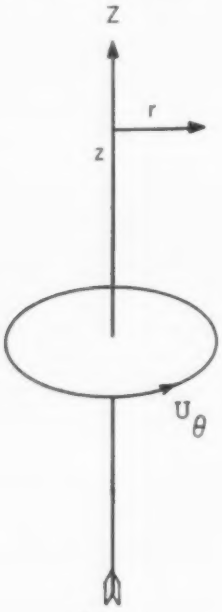


FIG. 1.

action. The purpose of the present paper is to extend the possibilities of the original model.

It is convenient, for simplicity, to confine attention to axially symmetrical problems, in which case the magnetic field and its associated vector potential may be produced by suitable distributions of circular currents about the axis of $z$. The magnetic field components $H_z$ and $H_r$ are related to the vector potential by the well-known relations

$$rH_z = \frac{\partial}{\partial r}(rU_\theta) \tag{1}$$

$$-rH_r = \frac{\partial}{\partial z}(rU_\theta). \tag{2}$$

### THE FUNDAMENTAL DYNAMICS OF THE PROBLEM

The motion of the particle is described by Lagrange's equations,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) - \frac{\partial L}{\partial q_r} = 0 \tag{3}$$

with $q_r$ representing $r$, $z$, or $\theta$ in the Lagrangian function $L$ which, for zero scalar potential $\varphi$, assumes the well-known form,

$$L = -mc^2\left[1 - \frac{r^2\dot{\theta}^2}{c^2} - \frac{\dot{r}^2}{c^2} - \frac{\dot{z}^2}{c^2}\right]^{1/2} + \frac{e}{c}r\dot{\theta}U_\theta \tag{4}$$

where $m$ is the rest mass and $e$ the charge.

For convenience of description in this special presentation, we shall further simplify the problem to one where everything concerned is independent of $z$, and where $U_\theta$ is zero at $t = 0$, and the particle starts from rest at $t = 0$.

Let us write

$$W = T + mc^2 = \frac{mc^2}{\sqrt{1 - \beta^2}} \tag{5}$$

where $T$ is the kinetic energy, and $\beta = u/c$. For the equations of motion corresponding to the $r$ and $\theta$ coordinates, Lagrange's equations yield, for the $r$ coordinate,

$$\frac{1}{c^2}\frac{d}{dt}(W\dot{r}) - \left(\frac{Wr\dot{\theta}^2}{c^2} + \frac{e\dot{\theta}U_\theta}{c}\right) - \frac{e}{c}r\dot{\theta}\frac{\partial U_\theta}{\partial r} = 0 \tag{6}$$

and for the $\theta$ coordinate

$$r\dot{\theta}W = -ceU_\theta. \tag{7}$$

The well-known dynamical theorem

$$\frac{d}{dt}(T + e\varphi) = -\frac{\partial L}{\partial t}$$

assumes, for zero $\varphi$, the form

$$\frac{dW}{dt} = -\frac{e}{c} r\dot{\theta} U_\theta \tag{8}$$

which, in view of Eq. 7 becomes

$$\frac{dW^2}{dt} = e^2 \frac{\partial U_\theta^2}{\partial t} = e^2 \left[ \frac{dU_\theta^2}{dt} - \dot{r} \frac{\partial U_\theta^2}{\partial r} \right]. \tag{9}$$

The incorporation of Eq. 7 in Eq. 6 yields

$$\frac{1}{c^2} \frac{d}{dt}(W\dot{r}) + \frac{e^2}{W} U_\theta \frac{\partial U_\theta}{\partial r} = 0. \tag{10}$$

Hence

$$\frac{W^2}{c^2} \ddot{r} + \frac{\dot{r}W}{c^2} \frac{dW}{dt} + e^2 U_\theta \frac{\partial U_\theta}{\partial r} = 0$$

which, by use of Eq. 9, yields

$$\frac{W^2}{c^2} \ddot{r} = -e^2 U_\theta \left( \frac{\partial U_\theta}{\partial r} + \frac{\dot{r}}{c^2} \frac{\partial U_\theta}{\partial t} \right). \tag{11}$$

A further interesting relation may be derived as follows: We have

$$W = \frac{m^2 c^4}{1 - \dot{r}^2/c^2 - r^2\dot{\theta}^2/c^2}. \tag{12}$$

From Eq. 7, $r^2\dot{\theta}^2 = c^2 e^2 U_\theta^2 / W$. Incorporating this in Eq. 12, we readily find

$$W^2 = \frac{m^2 c^4 + e^2 U_\theta^2}{1 - \dot{r}^2/c^2}. \tag{13}$$

We shall not have occasion to use all of the foregoing relations. We have written them down for the purpose of providing a background of useful formulae in terms of which not only our immediate problems, but also other matters, may be discussed as occasion may arise.

### THE CONDITIONS FOR CIRCULAR ORBITS, OUTWARD SPIRALLING AND INWARD SPIRALLING

*Case 1, Where $\partial U_\theta/\partial r$ is Zero*

If $\partial U_\theta/\partial r$ is zero, Eq. 10 shows that $W\dot{r}$ is constant with time, so that if it is zero initially, as in the case we are considering, it is always zero, and therefore $\dot{r}$ is always zero since $W(= T + mc^2)$ is never zero. This is the case of circular orbits. It is not necessary that $\partial U_\theta/\partial r$ shall be zero everywhere in order that a circular orbit may be possible. If $\partial U_\theta/\partial r$ is zero at any particular value of $r$ then, at that value, a circular orbit is possible. The case of a circular orbit was the one discussed by the present writer in his first paper on this matter.[2] It was there shown that a circular orbit was possible for all values of $r$ if the $z$ component of the magnetic field varied inversely as $r$, which corresponds, through (1), to the case of $U_\theta$ independent of $r$.

*Case 2, Where $\partial U_\theta^2/\partial t$ and $\partial U_\theta^2/\partial r$ Are Both Positive*

In this case, suppose $\dot{r}$ should find itself with a positive value. Then Eq. 11 shows that $\ddot{r}$ would be negative, so that $\dot{r}$ would diminish until it became zero. Then, from Eq. 11, $\ddot{r}$ would still be negative so that $\dot{r}$ would become negative. It would never become positive again without passing through zero; and when it was zero, $\ddot{r}$ would again be negative, so that it could not proceed to acquire a positive value. Hence, under the conditions stated, the particle *would spiral inwards*.

*Case 3, Where $\partial U_\theta^2/\partial t$ Is Positive and $\partial U_\theta^2/\partial r$ Is Negative*

In this case, if $\dot{r}$ should find itself negative, $\ddot{r}$ would be positive, and would certainly continue positive until $\dot{r}$ was zero where it would still be positive so that $\dot{r}$ would become positive. $\dot{r}$ could never become negative again without passing through zero, and it could never pass through zero because at $\dot{r} = 0$, $\ddot{r}$ is positive. Hence $\dot{r}$ would remain positive, and the path would *spiral outwards*.

### CONSIDERATION OF SPECIAL PROBLEMS

The picture we shall discuss is one in which, at all points, $U_\theta$ increases with time to a maximum and then decays. To correspond to the sort of conditions holding in sunspots, the time for $U_\theta$ to reach its maximum is of the order of 10 days, that is, about $10^6$ seconds.

The line integral of $U_\theta$ around a circle of axial distance $r$ is equal to the total magnetic flux through the circle of radius $r$. That is,

$$2\pi r U_\theta = \pi r^2 H_m \tag{14}$$

where $H_m$ is the maximum average value of $H_z$ within $r$. Let us suppose that the maximum average value of $H_z$ within an axial distance

[2] *Phys. Rev.*, Vol. 43, p. 217 (1933); JOUR. FRANKLIN INST., Vol. 215, p. 273 (1933).

of $10^9$ cm. is 2000 gauss. Then for $U_{\theta m}$, the maximum value of $U_\theta$ at this axial distance $10^9$, we shall have

$$U_{\theta m} = \frac{10^9 \times 2000}{2} = 10^{12}. \tag{15}$$

We shall further, for convenience, confine our interest to $U_\theta$ of the form

$$U_\theta = 10^{12} \left(\frac{r_o}{r}\right)^n F(t) \tag{16}$$

where $n$ is a constant (not necessarily an integer) greater than unity, $F(t)$ is a function of time which rises from zero at $t = 0$ to a maximum value unity and then decays. This form of expression is consistent with $U_\theta$ having the value $10^{12}$ at $r = r_o (= 10^9)$ at the time when it attains its maximum. We shall see that different characteristics of the particle motion may be exhibited by choosing different values of the exponent $n$.

Now one of the most important of the relationships cited above is contained in Eq. 9, which tells us that if the particle moves so that it has no radial velocity,

$$\frac{dW^2}{dt} = e^2 \frac{dU_\theta^2}{dt}$$

so that

$$W^2 - mc^2 = e^2 U_\theta^2. \tag{17}$$

As we have seen, the condition that the particle shall have no radial velocity is that $\partial U_\theta / \partial r$ shall be zero. If this condition holds, then a circular orbit can be described at all values of $r$. If it holds for only particular values of $r$, then circular orbits can exist only for those values of $r$. The situation in which the condition holds for all values of $r$ is that corresponding to Eq. 16, with $n = 0$. This case has the advantage that as $U_\theta$ rises continually to its maximum value, $W$, through Eq. 13, rises in corresponding fashion so that the energy attains the maximum value corresponding to $3 \times 10^{14}$ e.v. for the case cited, the case for which the average magnetic field within a radius of $10^9$ cm. amounts to 2000 gauss. The time taken to acquire this energy would be of the order of $10^6$ seconds, if that is the time for the aforesaid magnetic field to rise to its maximum.

The disadvantage of the foregoing case lies in the fact that when decay of the magnetic field sets in, the energy already acquired starts to be lost and would indeed become completely lost by the time that $U_\theta$ had returned to zero. Of course, it is possible to think of processes by which the charged particles became scattered, or otherwise deviated from their circular orbits in such manner that they were caused to leave

the region of potent acceleration and carry with them their energy. Indeed such procedures are actually employed in the betatron, but procedures of this kind are not so readily envisaged on a cosmical scale.

As we have already seen, with $U_\theta$ positive, if $\partial U_\theta/\partial r$ is positive, the particles will spiral inwards, and if it is negative, they will spiral outwards. Hence, outward spiralling always results for a vector potential of the type (16), if $n > 0$.

Outward spiralling is, in general, ineffective for the purpose of acquirement of high energy since the particle is taken off to regions where the rate of growth of energy is small before much energy has

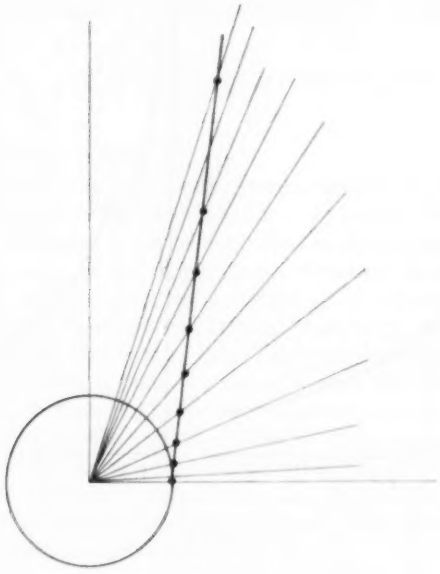

FIG. 2.

been acquired. An interesting case of this kind is one where the motivating currents are confined to circles about the axis of $z$ and lie within a radial distance $r_o$. The case is, in fact, that of an infinitely long solenoid. The peculiarity of this case is that there is no magnetic field outside the solenoid when the currents are steady; and even when the currents vary with time, the magnetic field outside $r_o$ is small. However, there is a very significant vector potential, given in fact at its maximum through Eqs. 14 and 15 and at $r = r_o$ by

$$U_{\theta m} = \frac{r_o}{2} H_m$$

where $H_m$ is the average flux per unit area within $r_o$, so that, with $r_o = 10^9$ and $H_m = 2000$

$$U_{\theta m} = 10^{12}.$$

At any value of $r$ greater than $r_o$, $U_\theta$ is consequently given by

$$U_\theta = 10^{12}(r_o/r)F(t).$$

The lines of the accelerating electric field are again circles around the axis of $z$, but there is no appreciable magnetic field for $r > r_o$.

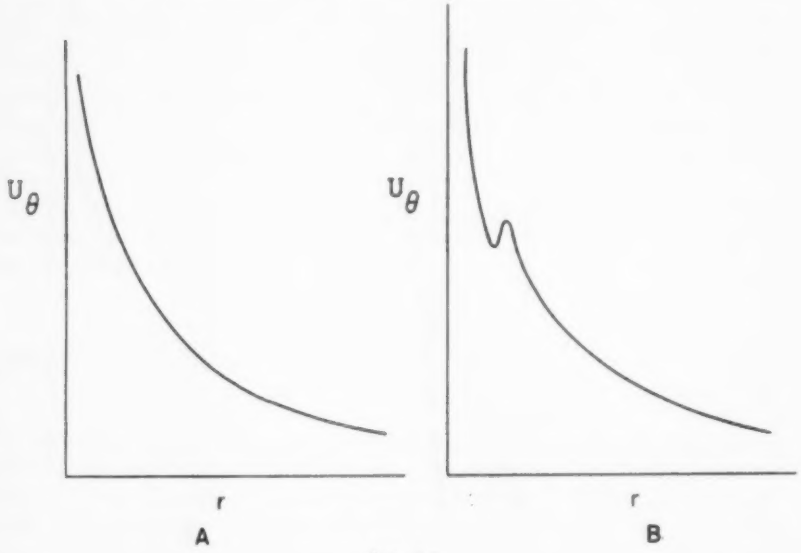


FIG. 3.

The solution for the orbit in this case can be carried out by numerical integration using Eqs. 7 and 11. The path is such that the particle starts out tangentially and curves rapidly into a path which is essentially straight and eventually becomes radial (see Fig. 2). The particle reaches a radial distance of about $5r_o$, in the first second of its motion. It is possible to show that at this stage it has acquired a radial velocity of $0.13c$ and a kinetic energy of $7 \times 10^7$ electron volts, most of this kinetic energy being in the radial velocity. The energy gained is only 4 per cent of the energy which would have been gained if the particle had moved in a circular orbit for one second at $r = r_o$.

As $n$ is reduced from unity, the particle develops a more pronounced outward spiral motion, the spiral loops being closer the smaller $n$, until finally, at $n = 0$, the particle describes a circular orbit.

However, in order to realize a circular orbit, it is not necessary to have a vector potential for which $\partial U_\theta/\partial r$ is zero *at all points*. Thus, suppose that, starting with a case where $n = 1$, so that $U_\theta$ is represented in Fig. 3*A*, we consider a modification like Fig. 3*B*, a case with a localized dip in $U_\theta$ at a certain value of $r$. Then not only will the particle be able to describe a closed circular orbit at the minimum of the dip, but since the distortion corresponds to a minimum, it follows immediately (from the arguments given for Cases 2 and 3) that if the particle is displaced outwards or inwards from the minimum, it will return to the minimum value, so that the distortion serves as a trap in which the particle can continue to circulate high energy. Thus we can readily show that such a trap is realized at $r = r_o$ if $U_\theta$ is of the form

$$U_\theta = B\left[\left(\frac{r_o}{r}\right)^n - \frac{n}{2\beta}\, e^{-\beta[(r/r_o)^2-1]}\right] F(t) \tag{18}$$

provided that $\beta < (n + 2)/2$.

Of course, such a trap calls for currents circulating in the region in such a manner as to be responsible for it. It is easy to show that the direction of such currents is opposite to that of the motivating currents, and indeed is the same as the direction provided for by the accelerating mechanism itself, which raises the question as to whether the particles which are themselves being accelerated to cosmic-ray energies may not provide the means by which they establish their own traps.

Of course, even with a trap, we are left with the original difficulty inherent in the fact that if the particle remains in the trap, while it will gain energy during the period in which $U_\theta$ increases with time, it will lose it again in the subsequent period when $U_\theta$ decreases with time. We can avoid this calamity by modifying Eq. 18 in such a manner that the two portions associated with it increase with time according to different laws. Thus we can write

$$U_\theta = B\left[\left(\frac{r_o}{r}\right)^n F(t) - \frac{n}{2\beta}\, e^{-\beta[(r/r_o)^2-1]} f(t)\right].$$

When we start to determine the conditions for a minimum of $U_\theta$, we find that, provided that the percentage time increase is greater for $F(t)$ than for $f(t)$, a trap starting at $r = r_o$ will move outwards with time provided that $(n + 2)/2 > \beta$, but it will only move out to a value given by $(r_m/r_o)^2 = (n + 2)/2\beta$. At this distance the trap disappears. The net result of all this is that the particles will circulate within the trap; and, on account of the very slow time changes of $F$ and $f$ associated with cosmical phenomena, will remain stable within the trap until it disappears, at which time they will spill over from the vanishing

trap and will proceed to describe paths which spiral outwards rapidly to infinity. Only during the time in which the particles remain in the trap will they be able to acquire significant energy; but this time may be made as long as we please by proper choice of the functions $F$ and $f$.

It is, of course, not to be supposed that nature stands still while the mathematician works out the kind of conditions he would like to have. However, in practice we are confronted with a situation in which the conditions described may arise automatically in different ways, and in different places, so that whenever they do arise, the conditions for acceleration to high energies prevail.

# SOME ENERGY RELATIONS FOR RC NETWORKS

BY

A. H. ZEMANIAN [1]

ABSTRACT

The class of RC two-port networks whose impulse responses contain no impulse functions is considered. Certain relations are established on some energy functions for these networks. The subclass of networks for which these relations assume an optimum form is then sought. It is found that this subclass consists of the compact RC networks whose open-circuit impedance parameters have residues that possess the same ratios at corresponding poles.

## INTRODUCTION

The purpose of this discussion is to establish some energy relations for RC networks and to determine a class of RC networks that is optimum in a certain sense. Although the RC networks considered in this paper are characterized in terms of their impedance parameters, the dual RL networks may be discussed in precisely the same way using the admittance parameters. Before restricting ourselves to RC networks, let us consider a general RLMC, two-port network having shunting resistance on both its input and output as shown in Fig. 1. The open-circuit impedance parameters of this network (including the input and output shunting resistances) will be designated by

$$z_{11} = F(s), \qquad z_{12} = z_{21} = W(s), \qquad z_{22} = G(s) \tag{1}$$

where $s$ is the complex frequency variable. It will be assumed henceforth that the asymptotic behavior of $F(s)$, $W(s)$, and $G(s)$, as $s$ goes to infinity, is given by

$$F(s) \sim \frac{K_1}{s}, \tag{2}$$

$$W(s) \sim \frac{K}{s^p}, \tag{3}$$

$$G(s) \sim \frac{K_2}{s}, \tag{4}$$

where $p$ is a positive integer. Such asymptotic behavior will always occur whenever the inevitable stray shunting capacitances occurring across the ports are taken into account. Furthermore, the values of

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$F(s)$, $W(s)$, and $G(s)$ at $s = 0$ will be denoted by $r_1$, $r$, and $r_2$, respectively. It follows that $r_1 \leq R_1$ and $r_2 \leq R_2$.

Now, if a unit impulse of current is impressed on the input at the time, $t = 0$, as shown in Fig. 1, the total energy dissipated in the resistor $R_2$ for all time is given by

$$\frac{1}{R_2}\int_0^\infty [w(t)]^2 dt \tag{5}$$

where $w(t)$ is the inverse Laplace transform of $W(s)$. Moreover, the total energy impressed into the network at the input is due solely to

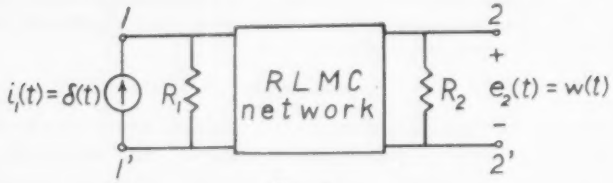


FIG. 1.

the one coulomb of charge injected into it at $t = 0$. At this instant the network behaves as a capacitor of $1/K_1$ farads. Hence, the energy impressed is $K_1/2$ joules. By the conservation of energy, we have

$$\int_0^\infty [w(t)]^2 dt \leq \frac{K_1 R_2}{2}. \tag{6}$$

Similarly, impressing a unit impulse of current on the output and invoking reciprocity, the same physical argument yields

$$\int_0^\infty [w(t)]^2 dt \leq \frac{K_2 R_1}{2}. \tag{7}$$

Taking the geometric mean, a result, which is weaker than the smaller of the two expressions (6) and (7), is obtained:

$$\int_0^\infty [w(t)]^2 dt \leq \tfrac{1}{2}[K_1 K_2 R_1 R_2]^{1/2}. \tag{8}$$

In the following analysis it will be shown that for RC networks (8) can be improved by replacing $R_1$ and $R_2$ by $r_1$ and $r_2$, respectively. Furthermore, given a realizable $w(t)$, the class of RC networks, for which the difference between the two sides of the relation corresponding to (8) is smallest, will be sought. In general, such an inequality does not hold for RLMC networks. For instance, consider the network shown in Fig. 2 and assume that $C = 1$ farad and that $R_1 = R_2 = 1$

ohm. If the inductance $L$ is indefinitely increased in value while the resistance $R$ is indefinitely decreased in value, the left-hand side of (8) approaches the numerical value of 1/4. On the other hand, $r = r_1 = r_2$ and these quantities approach zero. Hence, $\sqrt{K_1K_2r_1r_2}/2$ also approaches zero.

### ONE-PORT RC NETWORKS

Let $F(s)$ be the driving-point impedance of a one-port RC network, let $F(O) = r_1$, and let the asymptotic behavior of $F(s)$ as $s \to \infty$ be given by (2). (Here, we do not require that a single resistance across the port exists.) Thus, the unit impulse response $f(t)$ corresponding to $F(s)$ is given by

$$f(t) = \mathcal{L}^{-1}[F(s)] = \sum_{i=1}^{m} f_i e^{-\rho_i t} \tag{9}$$

where

$$\sum_{i=1}^{m} f_i = K_1, \qquad \sum_{i=1}^{m} \frac{f_i}{\rho_i} = r_1 \tag{10}$$

and where the real, negative, distinct constants, $-\rho_i$, represent the poles of $F(s)$ and the real, positive constants $f_i$ are the corresponding residues. We have

$$\int_0^\infty [f(t)]^2 dt = \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{f_i f_k}{\rho_i + \rho_k}. \tag{11}$$

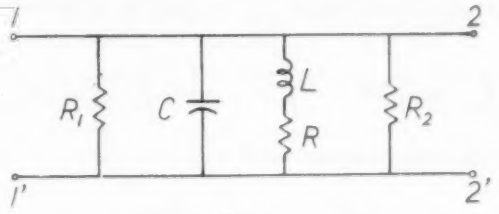


FIG. 2.

Using the inequality

$$\frac{2}{\rho_i + \rho_k} \leq \frac{1}{2\rho_i} + \frac{1}{2\rho_k}, \tag{12}$$

we obtain

$$\int_0^\infty [f(t)]^2 dt \leq \left[\sum_{i=1}^{m} f_i\right]\left[\sum_{i=1}^{m} \frac{f_i}{2\rho_i}\right] = \frac{K_1 r_1}{2}. \tag{13}$$

Equality cannot be achieved in (12) since $\rho_i \neq \rho_k$. Hence, equality occurs in (13) only when $m = 1$. Therefore, for the simple RC network shown in Fig. 3, the maximum energy, $K_1/2$, is dissipated in the shunting resistance $R_1$ when a unit impulse of current is applied. For any other RC one-port network, whose port terminals have a resistance

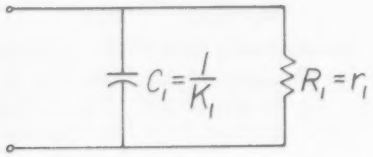


FIG. 3.

$R_1$ connected between them and whose impedance $F(s)$ has at least two poles and satisfies (2), the energy dissipated in $R_1$ will be less than $K_1 r_1/2R_1$.

### TWO-PORT RC NETWORKS

A similar result will now be established for two-port RC networks. We first wish to find a relation involving $w(t)$ that corresponds to (13). If the open-circuit impedance parameters satisfy (2), (3), and (4), the following inequality holds on the corresponding unit-impulse responses (1).[2]

$$\int_0^\infty [w(t)]^2 dt \leq \int_0^\infty f(t)g(t)dt. \tag{14}$$

Replacing the right-hand side of (14) by the use of Schwarz's inequality, we obtain

$$\int_0^\infty [w(t)]^2 dt \leq \left\{ \int_0^\infty [f(t)]^2 dt \int_0^\infty [g(t)]^2 dt \right\}^{1/2}. \tag{15}$$

Combining (15) with (13) and with

$$\int_0^\infty [g(t)]^2 dt \leq \frac{K_2 r_2}{2},$$

the desired result is obtained:

$$\int_0^\infty [w(t)]^2 dt \leq \tfrac{1}{2}[K_1 K_2 r_1 r_2]^{1/2}. \tag{16}$$

Furthermore, given a realizable $w(t)$, we wish to find all sets of $f(t)$, $w(t)$, and $g(t)$ that satisfy the necessary conditions for a two-port

[2] The boldface numbers in parentheses refer to the references appended to this paper.

RC network and for which the difference between the two sides of (16) attains its smallest magnitude. In addition to (9), we have

$$w(t) = \sum_{i=1}^{m} w_i e^{-\rho_i t}, \tag{17}$$

$$g(t) = \sum_{i=1}^{m} g_i e^{-\rho_i t}, \tag{18}$$

where $w_i$ and $g_i$ are the corresponding residues at each pole, $s = -\rho_i$. The residues are all real and satisfy the condition, $w_i^2 \leq f_i g_i$. Hence,

$$\int_0^\infty [w(t)]^2 dt = \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{w_i w_k}{\rho_i + \rho_k},$$

$$\int_0^\infty [w(t)]^2 dt \leq \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{[f_i g_i f_k g_k]^{1/2}}{\rho_i + \rho_k}. \tag{19}$$

The difference between the two sides of (19) is smallest when and only when the RC network is a compact one; that is, this occurs under the condition

$$w_i^2 = f_i g_i. \tag{20}$$

Hence, we have a physical means of interpreting the compact class of RC networks, as follows. Given a $W(s)$ that is realizable as an RC transfer impedance and satisfies (3), consider all corresponding sets of open-circuit impedance parameters $F(s)$, $W(s)$, and $G(s)$, that satisfy (2), (3) and (4). Among all such sets, it is the compact sets and only the compact sets for which the difference between the two sides of (19) is the smallest. We note that the left-hand side of (19) is proportional to the total energy dissipated in any resistor shunting the output port, if such a resistor exists. A simple computation shows that a similar remark may be made about $[w(t)]^2$, which is a quantity proportional to the instantaneous power dissipated in any terminating resistance. This quantity is also proportional to the instantaneous energy stored in any capacitor that is connected across the output.

Returning to the stated problem concerning (16), we first note that Schwarz's inequality for sums (2) states that, for $a_i$ and $b_i$ real,

$$\sum (a_i b_i)^{1/2} \leq [(\sum a_i)(\sum b_i)]^{1/2}.$$

So, for the right-hand side of (19), we have

$$\sum_{i=1}^{m} \sum_{k=1}^{m} \frac{[f_i g_i f_k g_k]^{1/2}}{\rho_i + \rho_k} \leq \left\{ \left[ \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{f_i f_k}{\rho_i + \rho_k} \right] \left[ \sum_{i=1}^{m} \sum_{k=1}^{m} \frac{g_i g_k}{\rho_i + \rho_k} \right] \right\}^{1/2}. \tag{21}$$

Equality occurs in the Schwarz inequality if and only if the $a_i$ and $b_i$ are proportional (2). Hence, equality occurs in (21) if and only if

$$\frac{f_i}{g_i} = k^2 \qquad (i = 1, 2, \cdots, m) \tag{22}$$

where $k^2$ is a constant. Using (12) and relations of the form (11) and (13) with the right-hand side of (21), (16) is again obtained. If $m \geq 2$, the set of inequalities corresponding to (12) cannot be improved since the $\rho_i$ are given and distinct. Finally, combining (20) and (22), we find that the ratios of the residues are the same positive constant $k$.

$$\frac{f_i}{|w_i|} = \frac{|w_i|}{g_i} = k \qquad (i = 1, 2, \cdots, m). \tag{23}$$

Under these conditions, both sides of (21) are independent of $k$ since the $w_i$ are given. Hence, we may conclude that the class of RC networks, for which (2), (3), and (4) are satisfied and for which the difference between the two sides of (16) has the least magnitude, is the class of compact RC networks that in addition have the property specified by (23).

The question remains whether such networks exist for all realizable $w(t)$. It is known that this class of networks is not void if $W(s)$ is a minimum-phase function. For, if $k = 1$, the impedance parameters are realizable (3).

E. S. Kuh (4) has obtained a similar result. For a voltage transfer ratio that is given except for its gain (that is, its constant multiplier), he has determined the admittance parameters of a three-terminal network needed to maximize the gain. Part of his solution states that the residues at all the poles of the admittance parameters must satisfy (23). Hence, the problem of maximizing the gain and the problem considered here lead to similar solutions.

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# PLASTIC INTERACTION CURVES FOR CIRCULAR CYLINDRICAL SHELLS UNDER COMBINED LATERAL AND AXIAL PRESSURES *

BY

R. SANKARANARAYANAN [1]

ABSTRACT

A circular cylindrical shell, made of a rigid perfectly plastic material, is subjected to a lateral pressure $P$ and an axial compressive pressure $Q$. The shell is clamped at one end and free at the other. The material of the shell is assumed to obey Tresca's yield condition and the associated flow rule. Plastic interaction curves based on a linearized yield surface are obtained for all values of $P$ and $Q$.

## 1. INTRODUCTION

Since there are many practical applications where circular cylindrical shells are subjected to combined lateral pressure $P$ and an axial pressure $Q$, it is of some interest to know the combination of $P$ and $Q$ for which plastic collapse of the shell will take place.

As a specific example we consider a shell clamped at one end and free at the other end as shown in Fig. 1. Similar solutions may be

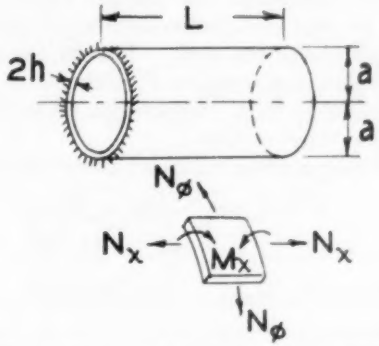


FIG. 1. Dimensions of circular cylindrical shell.

obtained for shells with different boundary conditions. The shell is subjected to a uniform lateral pressure $P$ and to an axial pressure $Q$.

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The material of the shell is rigid perfectly plastic and satisfies Tresca's yield condition and the associated flow rule.

The yield-point load of the shell is taken as a measure of its load-carrying capacity. This load is defined for any given ratio of $P/Q$ as that magnitude for which a rigid perfectly plastic shell can undergo a small quasi-static deformation, while remaining in equilibrium. It should be noted that the real shell may undergo elastic and elastic-plastic deformations for loads less than the yield-point load. These deformations, as well as deformations at the yield-point load, will change the geometry of the shell. This, in turn, will affect the carrying capacity. However, despite the deviations of the analysis from reality, it is hoped that the yield-point load found in this report will provide a meaningful estimate of the true carrying capacity.

The basic equations are given in the next section. Section 3 gives the details of the solution. Results for typical cases are shown graphically in the form of interaction curves for the pressures $P$ and $Q$.

## 2. BASIC EQUATIONS

A circular cylindrical shell with the dimensions indicated in Fig. 1 is made of a rigid perfectly plastic material with a tensile yield stress $\sigma_o$. The bending moment per unit length produced by axial stress is $M_x$; the axial and circumferential forces per unit length are given by $N_x$ and $N_\varphi$, respectively. The corresponding curvature and strains are $K_x$, $\epsilon_x$ and $\epsilon_\varphi$. The axial and the inward radial displacements are denoted by $U$ and $W$, and the axial coordinate $X$ is measured from the clamped (or built-in) end of the shell. The shell is assumed to be subjected to a uniform external pressure $P$, acting over the lateral surface of the cylinder, and to an axial compressive pressure $Q$, uniformly distributed over the end cross section.

The following dimensionless variables prove convenient:

$$\begin{aligned} &x = X/L; \qquad c^2 = L^2/ah; \qquad w = W/a; \qquad u = U/L \\ &n_x = N_x/N_o = N_x/2\sigma_o h; \qquad m_x = M_x/M_o = M_x/\sigma_o h^2 \\ &n_\varphi = N_\varphi/N_o = N_\varphi/2\sigma_o h; \qquad p = pa/2\sigma_o h \\ &q = Qa/2\sigma_o h. \end{aligned} \tag{1}$$

In the framework of small-deformation theory of shells the governing equations of equilibrium (1)[2] become

$$n_x = -q/2, \qquad m_x''/2c^2 + n_\varphi + p = 0 \tag{2}$$

where primes denote differentiation with respect to $x$.

The yield surface for a shell of uniform thickness, the material of which satisfies Tresca's yield condition, has been described by Onat (2). Hodge (3) has approximated this yield surface by a polyhedron,

[2] The boldface numbers in parentheses refer to the references appended to this paper.

TABLE I.—*Yield Polyhedron.*

| Face | Equation | Strain-rate vector ($\dot{u}'$, $-\dot{w}$, $-\dot{w}''/2c^2$) |
|---|---|---|
| 1 | $-n_\varphi = 1$ | $\lambda(0, -1, 0)$ |
| 2 | $n_\varphi = 1$ | $\lambda(0, 1, 0)$ |
| 3 | $n_x - 2n_\varphi + m_x = 2$ | $\lambda(1, -2, 1)$ |
| 4 | $-n_x + 2n_\varphi - m_x = 2$ | $\lambda(-1, 2, -1)$ |
| 5 | $n_x - 2n_\varphi - m_x = 2$ | $\lambda(1, -2, -1)$ |
| 6 | $-n_x + 2n_\varphi + m_x = 2$ | $\lambda(-1, 2, 1)$ |
| 7 | $n_x - n_\varphi = 1$ | $\lambda(1, -1, 0)$ |
| 8 | $-n_x + n_\varphi = 1$ | $\lambda(-1, 1, 0)$ |
| 9 | $n_x - m_x = 1$ | $\lambda(1, 0, -1)$ |
| 10 | $-n_x + m_x = 1$ | $\lambda(-1, 0, 1)$ |
| 11 | $-n_x - m_x = 1$ | $\lambda(-1, 0, -1)$ |
| 12 | $n_x + m_x = 1$ | $\lambda(1, 0, 1)$ |

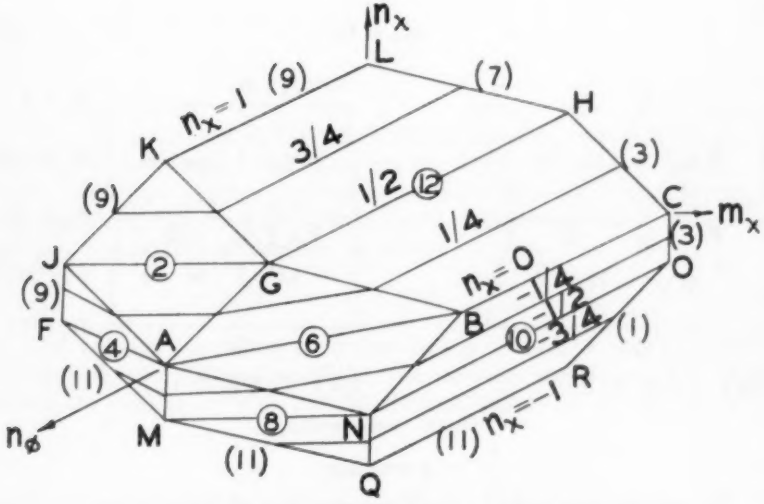


FIG. 2. Yield polyhedron for circular cylindrical shell with end load.

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bounded by twelve planes in the space of the stress resultants $(m_x, n_x, n_\varphi)$.

Table I which is reproduced[2] from (3) gives the equations of the planes, and Fig. 2 shows a part of the linearized yield surface. For each face of the polyhedron the strain-rate vector must be normal to that face; the corresponding strain-rate vector is given in Table I.

[2] Permission to reprint this material has been granted by the author (P. G. Hodge, Jr.) and the publisher. It originally appeared in Vol. 76 (pp. 336–342) of the *Transactions* of the American Society of Mechanical Engineers, in Vol. 21 of the *Journal of Applied Mechanics*.

At the intersection of two faces the strain-rate vector may be any positive combination of the vectors for the two faces forming the intersection.

A hinge circle can occur if the strain-rate vector corresponds to infinite $\dot{w}''$ and finite $\dot{w}$. Dots indicate that the quantities referred to are derivatives with respect to time. Evidently this can happen on any of the faces 9, 10, 11, 12, or on the edges joining those faces.

At the built-in end of the shell $x = 0$, the boundary conditions are

$$w = 0 \tag{3a}$$

and either (3)

$$w' = 0 \quad \text{or} \quad m_x + n_x = -1. \tag{3b}$$

At the free end $x = 1$, the shear and moment should vanish. Therefore,

$$m_x' = 0 \tag{3c}$$

and

$$m_x = 0. \tag{3d}$$

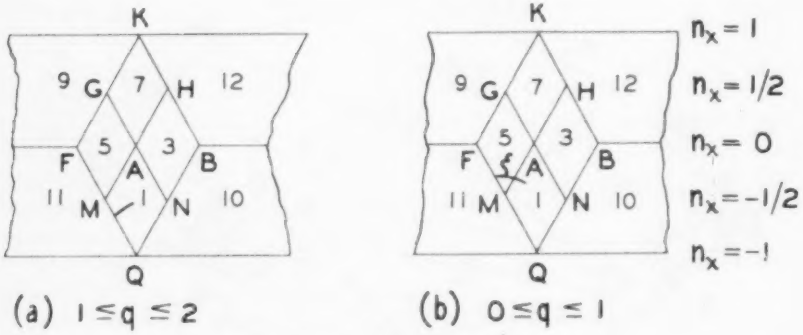


FIG. 3. Stress profiles.

### 3. SOLUTION

The general method of attack consists in making a hypothesis as to the "stress profile," that is, the locus of stress states in the space of the stress resultants involved. For any such hypothesis, the equilibrium equations and the flow rule, respectively, give the stress and velocity fields. The resulting "solution" must then be checked. It will be statically admissible if the entire stress profile lies on or inside the yield surface. It will be kinematically admissible if the strain-rate vector is always directed outward to the yield surface. Solutions which are statically or kinematically admissible will provide lower or upper bounds, respectively. The actual solution must be both statically and kinematically admissible.

From Eq. 2, it follows that axial equilibrium requires $n_x$ to be con-

stant. This fact simplifies the analysis of the problem, since it means that the entire stress profile lies on only one of the level curves shown in Fig. 2. From Fig. 2 it is also clear that the stress profile will correspond to different plastic regimes depending on whether $n_x$ is greater or less than $-\frac{1}{2}$. In order to make a choice between the two, we note that for pure axial compression ($q = 2$), $n_x = -1$. We therefore expect that for values of $q \approx 2$, $-1 \leq n_x \leq -\frac{1}{2}$. Moreover, we anticipate $n_\varphi$ to be negative.

Thus, we are led to the stress profile on face 1, as shown by the heavy line in Fig. 3*a*. From Table I, it follows that

$$n_\varphi = -1. \tag{4}$$

The substitution of Eq. 4 into Eq. 2 and integration subject to the boundary conditions (Eq. 3) result in

$$m_x = c^2(1 - p)(x - 1)^2; \qquad n_x = -\frac{q}{2} \tag{5a}$$

$$q + 2[c^2(P - 1) - 1] = 0. \tag{5b}$$

The above solution will be statically admissible, provided the stress profile lies everywhere within or on the yield surface. Reference to Table I and Eqs. 5 shows that the most restrictive inequalities are those corresponding to faces 5 and 11, and it is easily verified that the above solution is statically admissible, provided

$$1 \leq q \leq 2. \tag{6}$$

The velocity distribution corresponding to Eqs. 4 and 5 is easily determined from Table I and from the relevant equations (3). If we denote the velocity at $x = 1$ by $A$, the results can be written as

$$\dot{w} = Ax, \qquad \dot{u} = -\frac{A}{2c^2}. \tag{7}$$

It is seen that if $A \geq 0$, the velocity distribution satisfies all conditions for kinematic admissibility. Thus a yield-point load has been found which is associated with a statically admissible stress distribution, and with a kinematically admissible velocity distribution for values of $q$ such that $1 \leq q \leq 2$.

For values of $q \leq 1$, the inequality on face 5 is violated. We therefore assume the stress profile to start from face 1 for $x = 1$ and to terminate at the intersection of faces 5 and 11 for $x = 0$, with $x = \xi$ corresponding to the intersection of 1 and 5. The corresponding stress profile is shown in Fig. 3*b*. The solution under this hypothesis is easily worked out from Table I and from Eqs. 2 and 3. The results are

$$n_x = -\frac{q}{2}; \qquad n_\varphi = [2(p-1) - (2p-q-1)\cosh c\xi]\frac{\sinh cx}{2\sinh c\xi} + \tfrac{1}{2}(2p-q-1)\cosh cx - p;$$

$$m_x = [(2p-q-1)\cosh c\xi - 2(p-1)]\frac{\sinh cx}{\sinh c\xi} - (2p-q-1)\cosh cx + 2(p-1) - \frac{q}{2}; \qquad 0 \le x \le \xi \quad (8a)$$

$$\dot{w} = B\sinh cx; \qquad \dot{u} = \frac{B}{2c}[\cosh cx - 2];$$

$$n_x = -\frac{q}{2}; \qquad n_\varphi = -1; \qquad m_x = c^2(1-p)(x-1)^2$$

$$\dot{w} = B[c(x-\xi)\cosh c\xi + \sinh c\xi] \qquad \xi \le x \le 1$$

$$\dot{u} = \frac{B}{2c}[\cosh c\xi - 2] \quad (8b)$$

where $c$, $\xi$, $p$ and $q$ are related by the equations

$$q - 2c^2(p-1)(1-\xi)^2 = 0 \quad (9a)$$

$$2c(1-p)(\xi-1)\sinh c\xi - [(2p-q-1) - 2(p-1)\cosh c\xi] = 0. \quad (9b)$$

The strain-rate vectors associated with the velocity distribution (Eqs. 8*a*, *b*) are normal to the appropriate faces of the yield surface because of the method of their formation, and it is readily verified that the plastic multiplier $B$ is always positive. Equations 8, therefore, provide a kinematically admissible velocity field.

To determine the range of static admissibility of Eqs. 8, we note that for $\xi \le x \le 1$ the most restrictive inequalities are those corresponding to faces 5 and 11, and that the solution is statically admissible, provided

$$q \le 1. \quad (10)$$

With regard to the region $0 \le x \le \xi$, since $n_x$ is negative, the most restrictive inequalities are seen to be those corresponding to faces 1 and 11. In order to investigate the conditions under which these inequalities are satisfied, we use the following notations:

$$f(x) = m_x + 1 - \frac{q}{2} \quad (11a)$$

$$g(x) = n_\varphi + 1. \quad (11b)$$

Thus we have to find the conditions under which $f(x)$ and $g(x)$ are positive. Let us first confine our attention to $f(x)$. Substituting the expressions for $m_x$ and $n_\varphi$ from Eq. 8*a* into Eq. 11*a* and making use of Eq. 9*b*, we note that

$$f(0) = 0; \qquad f'(0) > 0. \tag{12a}$$

It is also seen that for values of $p$ and $q$ which satisfy the inequalities $p \geq 1$ and $q \leq 1$,

$$f(\xi) \geq 0; \qquad f'(\xi) \geq 0. \tag{12b}$$

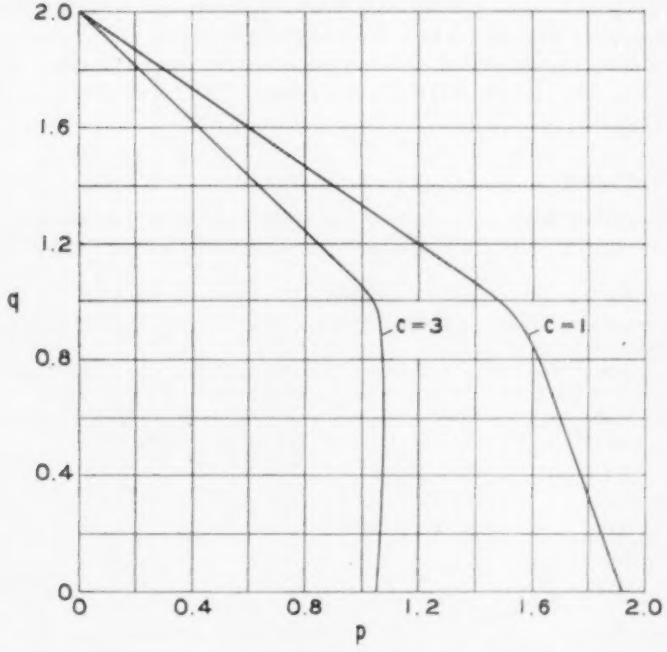


FIG. 4. Interaction curves for cylindrical shell under combined pressures.

In order to prove that $f(x) \geq 0$, we note that, in view of the Inequalities 12*a* and 12*b*, it is sufficient to show that the stationary values of the function $f(x)$, if any, occur outside the interval $0 \leq x \leq \xi$. Setting $f'(x) = 0$ (see Eqs. 8*a*, 9*b* and 11*a*),

$$\tanh cx_{ext} = \frac{(2p - q - 1)\cosh c\xi - 2(p - 1)}{(2p - q - 1)\sinh c\xi}. \tag{13}$$

Finally, for $x_{ext} \geq \xi$ we are led to the inequality

$$\coth c\xi - \tanh c\xi \geq \frac{2(p-1)}{(2p-q-1)} \operatorname{cosech} c\xi. \tag{14}$$

It is easily verified that for values of $\xi \geq 0$, Inequality 14 is always satisfied. In a similar manner, it can be shown that $g(x) \geq 0$. Thus the solution represented by Eqs. 8 is statistically and kinematically admissible for values of $q$ satisfying

$$0 \leq q \leq 1. \tag{15}$$

Equations 5*b* and 9 yield Fig. 4 which shows the resulting interaction curves for two different values of the geometric parameter $c$.

In a subsequent report, it is proposed to consider the effect of axial pressure on the plastic response of circular cylindrical shells under impulsive loading.

*Acknowledgment*

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# PURIFICATION FACTOR CHARACTERIZATION OF ZONE REFINING

BY

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ABSTRACT

A quantity $\pi_T$, termed the purification factor, defines the potential state of purity realizable in a specimen that has undergone zone-refinement treatment. An exact analysis appears intractable, but neglect of the terminal zone perturbation (which becomes vanishingly small for the infinite ingot) allows a lower bound to be established on the remanent solute impurity. Such numerical values of $\pi_T$ are evaluated for some sensible distribution coefficients and zone dimensions involving a maximum of ten passes. An original expression for the solute impurity distribution is deduced and numerically appraised for a range of operational conditions not presently tabulated; the relation has been applied to the indicated $\pi_T$ determinations.

## INTRODUCTION

The purpose here is to describe in some detail a useful parameter for characterizing the progress made at any stage of the zone-refining process. In doing so, it may also be of value to present still another manner for expressing the impurity distribution which evidently lends itself more readily to numerical calculation. Thus a quantity identified as a purification factor is explicitly evaluated for a range of conditions considered to be representative of zone-refining practice.

Actually, the present treatment is restricted to an upper bound identification of the degree of purification attained during the refinement precedure; analytic treatment of the terminal zone effect does not appear to be tractable, although the correction could be numerically evaluated. The technique of terminal zone cropping tends to moderate the departure from the ideal purification factor since it operates to offset the approach to an equilibrium impurity distribution. A brief description of this approach was offered earlier (1).[3]

## IMPURITY BALANCE AND THE PURIFICATION FACTOR

A simple impurity balance may be posed ignoring the terminal zone perturbation. Conservation of solute in an ingot of length $L$ having an initial distribution $C_0(x)$ may be represented as follows:

$$\sum S = C_0 L = S_T = S_{I,n} + S_{II,n} + S_{III,n-1} + \sum_{n-1} S_{f,n-1} \qquad (1)$$

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

where the total impurity solute $S_T$ is expressed in terms of the components $S_{I,n}$, the solute in the purified zone after the $n^{th}$ pass, $S_{II,n}$, the solute in the liquid zone, $S_{III,n-1}$, the solute in the remaining ingot from the previous pass, exclusive of the terminal zone, which must be summed over $S_{I,n-1}$ for the maximum accumulation of impurity after $n-1$ passes.

The compartmental distribution of impurity then is prescribed by the relations

$$S_{I,n} = \int_0^x C_n(x)dx \tag{2a}$$

$$S_{II,n} = \frac{l}{k} C_n(x) \tag{2b}$$

$$S_{III,n-1} = \int_{x+l}^{L-l} C_{n-1}(x)dx \tag{2c}$$

$$S_{f,n-1} = lC_{n-1}(x)\,|_{x=L-l} \tag{2d}$$

where $l$ is the zone length and $k$ is the solute distribution coefficient.

Thus for the fraction of solute impurity removed after the $n^{th}$ pass $P_n$ there derives

$$P_n = \frac{1}{C_0 L}\int_0^{L-l} [C_{n-1}(x) - C_n(x)]dx \tag{3}$$

whereas for the cumulative removal of impurity it follows that

$$P_T = \sum_n P_n = \left(C_0 L - \int_0^{L-l} C_n(x)dx\right)(C_0 L)^{-1}. \tag{4}$$

The fraction of impurity yet to be transferred to the terminal zone may be defined as the purification factor $\pi_T$ and is clearly the result

$$\pi_T = 1 - P_T. \tag{5}$$

Explicit calculation of the purification factor demands identification of $C_n(x)$ and the subsequent evaluation of Eq. 4; the next section deals with this aspect.

### ELABORATION OF THE $P_T$ RELATION[4]: A NEW FORM FOR $C_n(X)$

The detailed calculations of the purification factors are based upon a form of $C_n(x)$ not previously described in the literature (2). Starting with the underlying equation (3) which specifies the spatial distribution

[4] The usual idealizations are embodied as, for example, perfect mixing of components in the liquidus, limited diffusion in the solidus, etc.

of solute

$$\frac{dS_n(x)}{dx} = C_{(n-1)}(x) - \frac{k}{l} S_n(x) \tag{6}$$

the solution may readily be shown to be of the form

$$S_n(x) = e^{-(k/l)x} \left\{ \int_0^x C_{(n-1)} e^{(k/l)x} dx + K_{(n-1)} \right\} \tag{7}$$

wherein the $K_{(n-1)}$ are integration constants prescribed by the initial condition

$$C_n(x)\,|_{x=0} = k\bar{c}, \qquad \bar{c} = \frac{1}{l}\int_0^l C_{(n-1)}(x)dx \tag{8}$$

when utilized in conjunction with

$$C_n(x) = \frac{k}{l} S_n(x). \tag{9}$$

In the appendix are provided details for the inductive prescription of $Q_n(x)$ in the generalized form of $C_n(x)$ which resembles that of the single pass result:

$$C_n(x)/C_0 = 1 - Q_n(1-k)e^{-(k/l)x} \tag{10a}$$

with

$$Q_n = \sum_{p=n}^{1} A_p \frac{\left(\frac{k}{l}x\right)^{n-p}}{(n-p)!} \tag{10b}$$

and the diverse $A_p$ in turn given by

$$A_j = 1 - \frac{A_{j-1}S_1}{0!} - \frac{A_{j-2}S_2}{1!} - \frac{A_{j-3}S_3}{2!} - \cdots$$
$$- \frac{A_2 S_{j-2}}{(j-3)!} + \frac{1}{(j-2)!}\left(\frac{k}{l}\right)^{j-1} \int_0^l x^{j-2} e^{-(k/l)x} dx. \tag{10c}$$

In (10c) the $S_m$ takes on values determined from

$$S_m = -\int_0^k x^{j-1} e^{-x} dx = -\left[\int k^{j-1} e^{-k} dk + (j-1)!\right]. \tag{10d}$$

Typically, for $j = 1$, $A_1 = 1$ and $Q_1 = 1$.

In Table I are collected the solute distributions $C_n(x)/C_0$ as a function of $x/l$ for $n = 1$ to 10 with $k$ values selected as 0.001, 0.007, 0.05, 0.07 and 0.12. The over-all behavior of the solute distribution is exhibited in Figs. 1–6; two different ranges are displayed in Figs. 1 and 2, with the latter appropriate for small $x/l$ and $k = 0.001$. Other values

TABLE I.—$C_n(x)/C_0$ vs x/l *Data for* k = 0.001, 0.007, 0.05, 0.07, 0.12.

$k = .001$

| $\frac{x}{l}$ | $\frac{C_1(x)}{C_0}$ | $\frac{C_2(x)}{C_0}$ | $\frac{C_3(x)}{C_0}$ | $\frac{C_4(x)}{C_0}$ | $\frac{C_5(x)}{C_0}$ | $\frac{C_6(x)}{C_0}$ | $\frac{C_7(x)}{C_0}$ | $\frac{C_8(x)}{C_0}$ | $\frac{C_9(x)}{C_0}$ | $\frac{C_{10}(x)}{C_0}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .001 | $.150 \cdot 10^{-5}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | .001985 | $.300 \cdot 10^{-5}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | .01094 | $.610 \cdot 10^{-4}$ | $.23 \cdot 10^{-6}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100 | .09607 | .004771 | $.1593 \cdot 10^{-5}$ | $.400 \cdot 10^{-5}$ | $.8 \cdot 10^{-7}$ | 0 | 0 | 0 | 0 | 0 |
| 500 | .3935 | .09051 | .01446 | .001764 | $.1757 \cdot 10^{-3}$ | $.143 \cdot 10^{-4}$ | $.102 \cdot 10^{-5}$ | $.6 \cdot 10^{-7}$ | 0 | 0 |
| 1000 | .6321 | .2646 | .08048 | .01574 | .003675 | $.5973 \cdot 10^{-3}$ | $.838 \cdot 10^{-4}$ | $.103 \cdot 10^{-4}$ | $.113 \cdot 10^{-5}$ | $.11 \cdot 10^{-6}$ |
| 2000 | .8647 | .5943 | .3236 | .1418 | .05274 | .01660 | .005050 | .001100 | $.2382 \cdot 10^{-3}$ | $.470 \cdot 10^{-4}$ |
| 3000 | .9502 | .8010 | .5770 | .3530 | .1849 | .08402 | .03356 | .01193 | .003811 | .001105 |
| 4000 | .9817 | .9085 | .7620 | .5667 | .3714 | .2150 | .1108 | .05119 | .02139 | .008146 |
| 5000 | .9913 | .9596 | .8754 | .7351 | .5597 | .3842 | .2380 | .1335 | .06816 | .03186 |
| 6000 | .9975 | .9827 | .9381 | .8489 | .7151 | .5545 | .3938 | .2526 | .1529 | .08399 |
| 7000 | .9991 | .9927 | .9704 | .9183 | .8271 | .6994 | .5504 | .4014 | .2710 | .1696 |
| 8000 | .9997 | .9970 | .9862 | .9576 | .9004 | .8088 | .6867 | .5472 | .4076 | .2835 |
| 9000 | .9999 | .9988 | .9938 | .9788 | .9451 | .8845 | .7933 | .6763 | .5445 | .4127 |
| 10,000 | 1.0 | .9995 | .9972 | .9897 | .9708 | .9330 | .8699 | .7799 | .6670 | .5422 |
| 12,000 | | | | | | | | .9105 | .8450 | .7577 |
| 15,000 | 1.0 | 1.0 | 1.0 | .9998 | .9991 | .9972 | .9924 | .9820 | .9626 | .9302 |
| 20,000 | | | | 1.0 | 1.0 | .9999 | .9997 | .9992 | .9979 | .9950 |
| 22,000 | | | | | | 1.0 | 1.0 | .9998 | .9994 | .9988 |

$k = .007$

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | .007 | .0000733 | .00000074 | | | | | | | |
| 1 | .01393 | .0001458 | .000003 | 0 | | | | | | |
| 3 | .02764 | .000434 | | 0 | | | | | | |
| 10 | .07413 | .002864 | .000073 | .0000018 | 0 | 0 | | | | |
| 30 | .1951 | .02044 | .001461 | | 0 | 0 | 0 | | | |
| 50 | | | | .000512 | .0000366 | .0000025 | 0 | 0 | 0 | 0 |
| 100 | .5069 | .1583 | .03502 | .00599 | .00090 | .000074 | .0000083 | 0 | 0 | 0 |
| 200 | .7551 | .4106 | .1682 | .05456 | .01432 | .00343 | .000703 | .00016 | .0000172 | .00000264 |
| 300 | .8784 | .6222 | .3523 | .1627 | .06217 | .02078 | .0060 | .00156 | .000346 | .0000718 |
| 500 | .9700 | .8648 | .6705 | .4649 | .2746 | .1434 | .0659 | .02707 | .01004 | .00336 |
| 600 | | | | | | .2480 | .1334 | .06448 | .02825 | .01132 |
| 800 | | | .9180 | .8101 | .6579 | .4893 | .3309 | .2040 | .1150 | .05956 |
| 1000 | .9991 | .9928 | .9710 | .9186 | .8206 | .7002 | .5514 | .4024 | .2718 | .1703 |
| 1300 | | | | | | .8907 | .8030 | .6885 | .5584 | .4267 |
| 1500 | 1.0 | .9997 | | .9929 | .9789 | .9498 | .8987 | .8220 | .7213 | .6037 |
| 2000 | | 1.0 | .9999 | .9995 | .9982 | .9945 | .9858 | .9685 | .9382 | .8909 |
| 2500 | | | 1.0 | | | | | .9960 | .9906 | .980 |
| 3000 | | | | 1.0 | 1.0 | 1.0 | .9999 | .9996 | .999 | .9972 |
| 4000 | | | | | | | | | | 1.0 |

TABLE I.—*Continued.*

| $\frac{x}{l}$ | $\frac{C_1(x)}{C_0}$ | $\frac{C_2(x)}{C_0}$ | $\frac{C_3(x)}{C_0}$ | $\frac{C_4(x)}{C_0}$ | $\frac{C_5(x)}{C_0}$ | $\frac{C_6(x)}{C_0}$ | $\frac{C_7(x)}{C_0}$ | $\frac{C_8(x)}{C_0}$ | $\frac{C_9(x)}{C_0}$ | $\frac{C_{10}(x)}{C_0}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $k = .05$ | | | | | |
| 0 | .05 | .003668 | .000260 | .0000184 | .0000013 | .0000001 | 0 | | | |
| 1 | .0963 | .007076 | .000501 | .0000354 | .0000025 | .0000002 | 0 | | | |
| 4 | .2222 | .02853 | .002780 | .000229 | .0000172 | .0000013 | .0000001 | 0 | | |
| 10 | .4238 | .1076 | .01945 | .002751 | .000323 | .0000330 | .0000030 | .0000002 | 0 | |
| 15 | .5512 | .1928 | .04857 | .009541 | .001539 | .000212 | .0000258 | .0000027 | .0000003 | 0 |
| 20 | .6505 | .2840 | .09094 | .02283 | .004706 | .000824 | .000125 | .0000169 | .0000021 | .0000002 |
| 30 | .7880 | .4597 | .2050 | .07293 | .02146 | .005372 | .001169 | .000225 | .0000388 | .0000061 |
| 40 | .8714 | .6080 | .3379 | .1530 | .05790 | .01875 | .005295 | .001324 | .000297 | .0000604 |
| 50 | .9220 | .7233 | .4698 | .2541 | .1164 | .04591 | .01586 | .004866 | .001341 | .000335 |
| 60 | .9527 | .8085 | .5886 | .3648 | .1940 | .08964 | .03645 | .01320 | .004302 | .001274 |
| 80 | .9826 | .9122 | .7695 | .5768 | .3817 | .2234 | .1165 | .0545 | .0233 | .00892 |
| 100 | .9936 | .9613 | .8797 | .7423 | .5688 | .3935 | .2458 | .1392 | .0718 | .0339 |
| 120 | .9976 | .9834 | .9403 | .8534 | .7220 | .5629 | .4024 | .2635 | .1585 | .0878 |
| 140 | .9991 | .9930 | .9715 | .9209 | .8318 | .7060 | .5582 | .4093 | .2780 | .1751 |
| 160 | .9997 | .9971 | .9868 | .9591 | .9033 | .8136 | .6931 | .5545 | .4150 | .2901 |
| 180 | .9999 | .9988 | .9940 | .9795 | .9468 | .8875 | .79800 | .6823 | .5514 | .4197 |
| 200 | 1.0 | .9995 | .9973 | .9900 | .9717 | .9349 | .8732 | .7845 | .6732 | .5488 |
| 220 | 1.0 | .9998 | .9988 | .9953 | .9854 | .9636 | .9235 | .8602 | .7727 | .6653 |
| 240 | | .9999 | .9995 | .9978 | .9927 | .9803 | .9555 | .9128 | .8484 | .7622 |

TABLE 1—*Continued.*

| $\frac{x}{i}$ | $\frac{C_1(x)}{C_0}$ | $\frac{C_2(x)}{C_0}$ | $\frac{C_3(x)}{C_0}$ | $\frac{C_4(x)}{C_0}$ | $\frac{C_5(x)}{C_0}$ | $\frac{C_6(x)}{C_0}$ | $\frac{C_7(x)}{C_0}$ | $\frac{C_8(x)}{C_0}$ | $\frac{C_9(x)}{C_0}$ | $\frac{C_{10}(x)}{C_0}$ | $\frac{C_{11}(x)}{C_0}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $k = .07$ | | | | | |
| 0 | .07 | .007126 | .0007007 | .00006879 | .0000068 | .0000007 | 0 | 0 | 0 | 0 | 0 |
| 1 | .1329 | .01355 | .001333 | .0001308 | .00001285 | .0000013 | .0000001 | 0 | 0 | 0 | 0 |
| 3 | .2462 | .03689 | .004352 | .0004585 | .00004608 | .0000046 | .0000004 | 0 | 0 | 0 | 0 |
| 5 | .3446 | .07096 | .01078 | .001355 | .0001514 | .00001582 | .0000016 | 0 | 0 | 0 | 0 |
| 8 | .4688 | .1354 | .02830 | .004701 | .0006603 | .00008202 | .0000093 | 0 | 0 | 0 | 0 |
| 10 | .5382 | .1837 | .04548 | .008885 | .001448 | .0002051 | .00002604 | .0000030 | .0000004 | 0 | 0 |
| 15 | .6746 | .3108 | .106091 | .02859 | .006370 | .001214 | .0002029 | .00003041 | .0000042 | 0 | 0 |
| 20 | .7707 | .4341 | .1861 | .06360 | .01801 | .004354 | .0009189 | .0001724 | .00002921 | .00000452 | .00000065 |
| 30 | .8861 | .6393 | .3712 | .1767 | .07063 | .02423 | .007265 | .001934 | .0004628 | .0001006 | .00002007 |
| 40 | .9434 | .7813 | .5485 | .3255 | .1650 | .07250 | .02801 | .009634 | .002983 | .0008397 | .0002166 |
| 50 | .9719 | .8717 | .6929 | .4799 | .2894 | .1532 | .07178 | .03009 | .01139 | .003928 | .0012419 |
| 60 | .9860 | .9265 | .7995 | .6185 | .4252 | .2598 | .1418 | .06969 | .03103 | .01262 | .004713 |
| 70 | | | | | | .3802 | .2348 | .1313 | .06689 | .03120 | .01341 |
| 80 | .9966 | .9771 | .9218 | .8174 | .6693 | .5012 | .3422 | .2132 | .1216 | .06373 | .03085 |
| 100 | .9992 | .9932 | .9720 | .9220 | .8338 | .7089 | .5617 | .4129 | .2812 | .1776 | .1043 |
| 110 | | | | | .8868 | .7874 | .6586 | .5155 | .3765 | .2563 | .1628 |
| 120 | .9998 | .9980 | .9905 | .9694 | .9245 | .8486 | .7413 | .6114 | .4738 | .3442 | .2343 |
| 130 | .9999 | .9989 | .9946 | .9812 | .9507 | .8946 | .8088 | .6964 | .5674 | .4360 | .3154 |
| 140 | | | | | | .9280 | .8618 | .7684 | .6531 | .5266 | .4017 |
| 150 | | | | | .9799 | .9518 | .9022 | .8273 | .7282 | .6119 | .4888 |
| 160 | | | | | | | .9320 | .8737 | .7916 | .6888 | .5728 |
| 180 | 1.0 | 1.0 | .9997 | .9986 | .9953 | .9868 | .9687 | .9361 | .8845 | .8117 | .7198 |
| 200 | | 1.0 | .9999 | .9996 | .9983 | .9947 | .9864 | .9697 | .9402 | .8941 | .8293 |
| 220 | | 1.0 | | .9999 | .9994 | .9980 | .9944 | .9863 | .9708 | .9442 | .9029 |
| 250 | | | | | | .9996 | .9986 | .9962 | .9909 | .9807 | .9626 |

TABLE I.—*Continued.*

| $\frac{x}{l}$ | $\frac{C_1(x)}{C_0}$ | $\frac{C_7(x)}{C_0}$ | $\frac{C_8(x)}{C_0}$ | $\frac{C_9(x)}{C_0}$ | $\frac{C_5(x)}{C_0}$ | $\frac{C_6(x)}{C_0}$ | $\frac{C_7(x)}{C_0}$ | $\frac{C_8(x)}{C_0}$ | $\frac{C_9(x)}{C_0}$ | $\frac{C_{10}(x)}{C_0}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $k = .12$ | | | | | |
| 0 | .12 | .02049 | .003386 | .0005588 | .00009223 | .00001522 | .0000025 | 0 | 0 | 0 |
| 1 | .3132 | .03759 | .006214 | .001025 | .0001692 | .00002794 | .0000046 | .0000008 | 0 | 0 |
| 2 | .3908 | .06335 | .01118 | .001875 | .0003104 | .00005125 | .0000085 | .0000014 | 0 | 0 |
| 3 | .4597 | .09560 | .01888 | .003343 | .0005649 | .00009387 | .00001552 | .0000026 | 0 | 0 |
| 4 | .5208 | .1325 | .02965 | .005692 | .001005 | .0001703 | .00002513 | .0000047 | .0000008 | 0 |
| 5 | .5750 | .1727 | .04358 | .009175 | .001724 | .0003031 | .00005142 | .0000086 | .0000014 | 0 |
| 6 | .6230 | .2148 | .06059 | .01402 | .002832 | .0005234 | .00009159 | .00001554 | .0000026 | 0 |
| 7 | .6657 | .2580 | .08051 | .02041 | .004450 | .0007711 | .0001591 | .00002771 | .0000047 | .0000008 |
| 8 | .7035 | .3015 | .1031 | .02848 | .006704 | .001400 | .0002683 | .00004839 | .0000084 | .0000014 |
| 9 | .7370 | .3446 | .1280 | .03834 | .009718 | .002166 | .0004379 | .00008237 | .00001472 | .0000025 |
| 10 | .7668 | .3869 | .1550 | .05001 | .01361 | .003236 | .0006918 | .0001363 | .00002526 | .0000045 |
| 20 | .9202 | .7195 | .4664 | .2525 | .1162 | .04634 | .01629 | .005120 | .001459 | .0003811 |
| 30 | .9788 | .8867 | .7206 | .5143 | .3215 | .1771 | .08679 | .03818 | .01521 | .005540 |
| 40 | .9936 | .9572 | .8697 | .7261 | .5490 | .3743 | .2303 | .1284 | .06527 | .03042 |
| 50 | .9981 | .9845 | .9437 | .8605 | .7330 | .5767 | .4168 | .2765 | .1686 | .09487 |
| 75 | .9999 | .9989 | .9944 | .9807 | .9494 | .8923 | .8055 | .6922 | .5629 | .4316 |
| 100 | 1.0 | .9999 | .9995 | .9979 | .9931 | .9813 | .9575 | .9163 | .8538 | .7696 |
| 125 | | 1.0 | 1.0 | .9998 | .9992 | .9975 | .9930 | .9834 | .9651 | .9345 |
| 150 | | | | 1.0 | .9999 | .9997 | .9990 | .9973 | .9935 | .9857 |
| 175 | | | | | 1.0 | 1.0 | .9999 | .9996 | .9990 | .9974 |

of $k$, ranging from 0.007, 0.05, 0.07 to 0.12 are displayed successively in Figs. 3, 4, 5, and 6.

These results of $C_n(x)/C_0$ are restricted to the infinite bar (that is, terminal zone freezing neglected) and supplement data now incorporated as standard matter (2). The validity of Eqs. 10 is thus established.

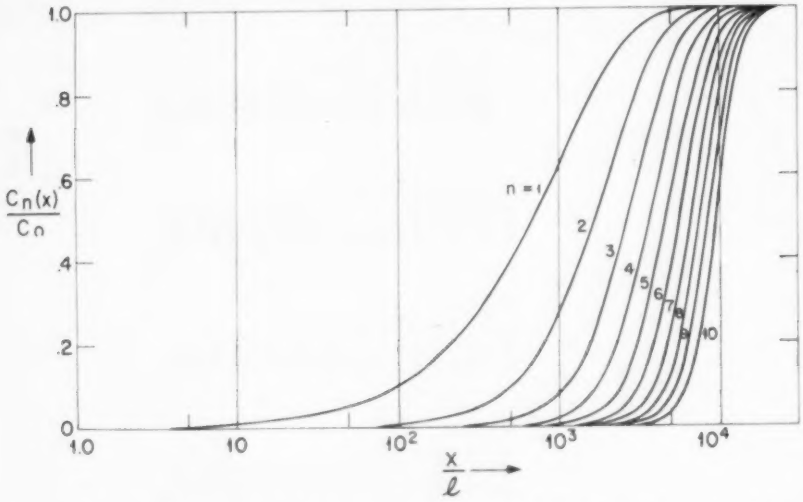


FIG. 1. Transformation of the solute distribution $C_n(x)/C_0$ by cascade zone refining. The parameter $n$ indicates stage of purification. The case $k = 0.001$ showing approach to a limiting distribution as $n \to \infty$.

Incorporating the new findings in Eq. 4 produces

$$P_T = \frac{l}{L} + \frac{(1-k)}{L}\int_0^{L-l} e^{-(k/l)x} \cdot \sum_{p=n}^{1} A_p \frac{\left(\frac{k}{l}x\right)^{n-p}}{(n-p)!}\,dx \qquad (11)$$

whence the purification factor may be written in the form

$$\pi_T = 1 - \frac{l}{L} + \frac{(1-k)}{L}\cdot\frac{l}{k}\left[\frac{A_nB_1}{0!} + \frac{A_{n-1}B_2}{1!} + \frac{A_{n-2}B_3}{2!} + \cdots \frac{A_1B_n}{(n-1)!}\right] \qquad (12a)$$

wherein the $A_n$ are as stated in Eq. 10$c$ and now

$$B_n = \left(\frac{k}{l}\right)^n \int_0^{L-l} x^{n-1}e^{-(k/l)x}dx, \qquad (12b)$$

## CONCLUSION

The paramount objective of characterizing the zone-refinement operation by introduction of a purification factor has now been fulfilled. The investigation may be appreciated more fully by numerical

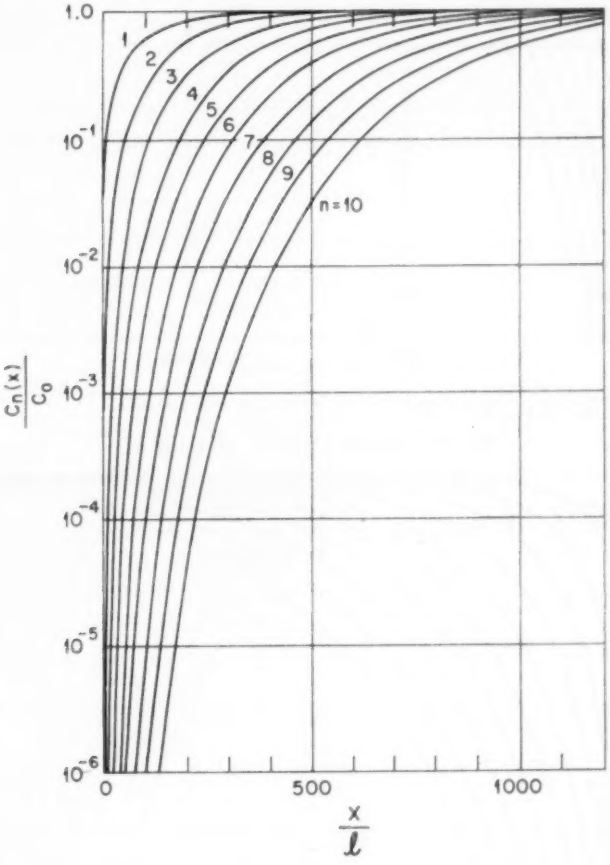


FIG. 2. The case $k = 0.001$ in the region of $\frac{x}{l} \to 0$. (Cf. Fig. 1.)

description of $\pi_T$ for the values of $k$ already embodied in the delineation of the solute distribution. Table II contains the $\pi_T$ behavior for three selected values of $l/L$, *viz.* 1/2, 1/4 and 1/10. Finally, Figs. 7, 8 and 9 depict the expected fall-off of the purification factor with increasing passes for the gamut of $k$ and $l/L$ indicated.

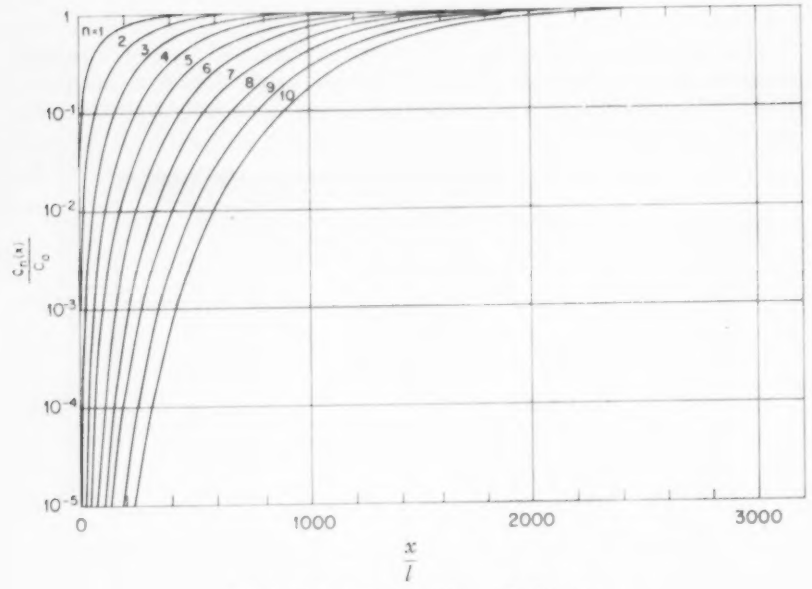


FIG. 3. The case $k = 0.007$. (Cf. Fig. 1.)

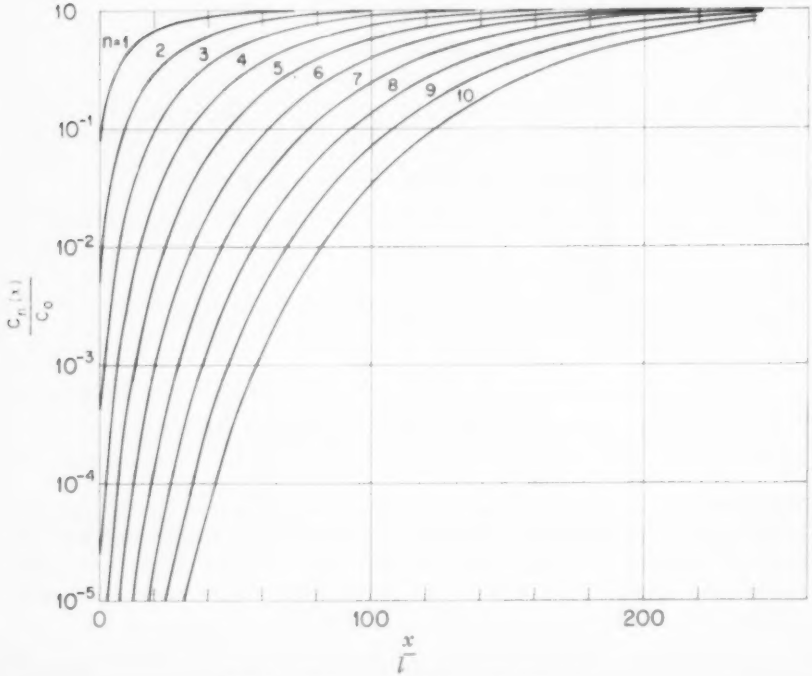


FIG. 4. The case $k = 0.05$. (Cf. Fig. 1.)

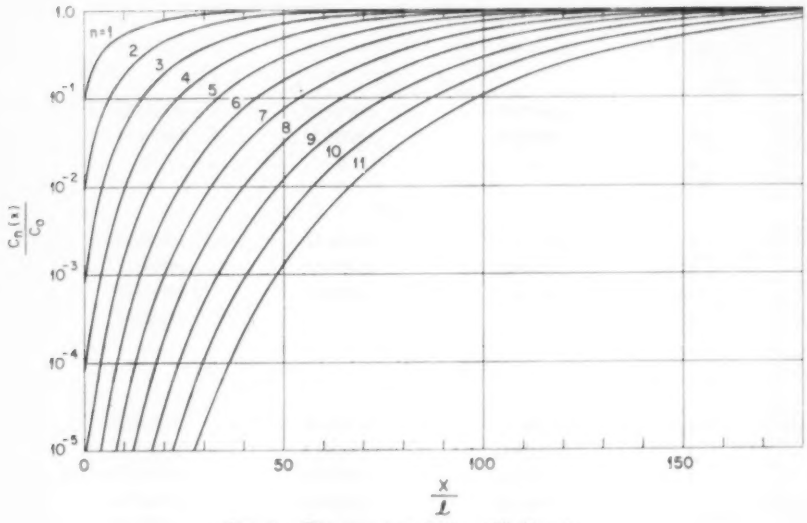


FIG. 5. The case $k = 0.07$. (Cf. Fig. 1.)

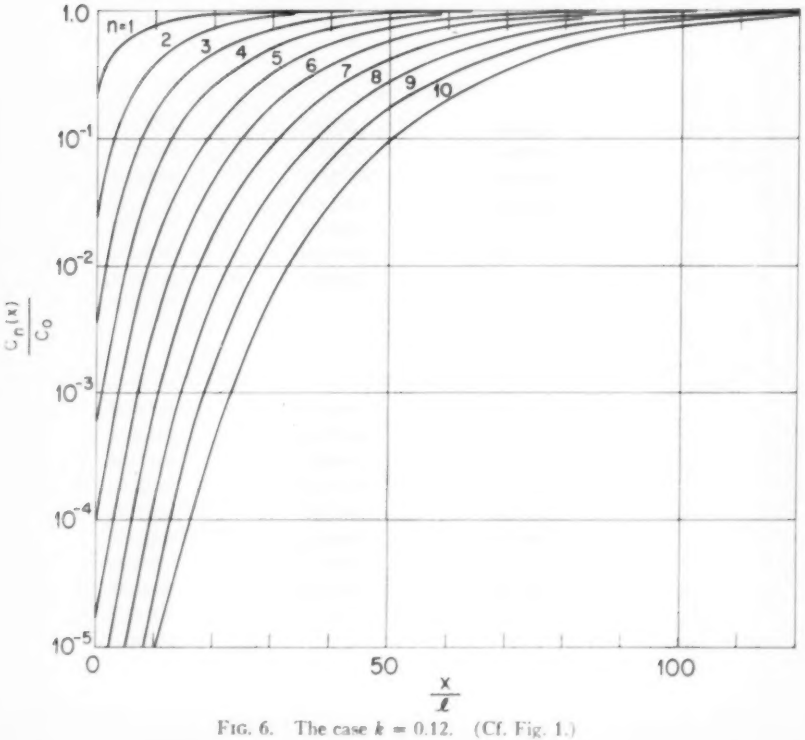


FIG. 6. The case $k = 0.12$. (Cf. Fig. 1.)

TABLE II.—$\pi_T$ vs n *Data for* I/L = 0.5, 0.25, 0.10.

| $n$ | $I/L = 0.5$ | $I/L = 0.25$ | $I/L = 0.10$ |
|---|---|---|---|
| | | $k = .001$ | |
| 1 | .0007496 | .0018728 | .0058298 |
| 2 | .0000010 | .0000034 | .0000174 |
| 3 | 0 | 0 | 0 |
| | | $k = .007$ | |
| 1 | .0052337 | .0130154 | .0338696 |
| 2 | .0000528 | .0001633 | .00083117 |
| 3 | .0000005 | .0000018 | .0000156 |
| 4 | 0 | 0 | .0000002 |
| 5 | | | 0 |
| | | $k = .05$ | |
| 1 | .0366795 | .0883629 | .2114935 |
| 2 | .0025966 | .0077118 | .0345802 |
| 3 | .0001836 | .0005898 | .0044488 |
| 4 | .0000130 | .0000428 | .0004804 |
| 5 | .0000009 | .0000030 | .0000455 |
| 6 | 0 | .0000002 | .0000039 |
| 7 | | 0 | .0000003 |
| 8 | | | 0 |
| | | $k = .07$ | |
| 1 | .0509018 | .1208691 | .2790148 |
| 2 | .0050049 | .0145876 | .0618266 |
| 3 | .0004914 | .9015478 | .0109057 |
| 4 | .0000483 | .0001559 | .0016245 |
| 5 | .0000047 | .0000154 | .0002129 |
| 6 | .0000005 | .0000015 | .0000254 |
| 7 | 0 | .0000001 | .0000028 |
| 8 | | 0 | .0000003 |
| 9 | | | 0 |
| | | $k = .12$ | |
| 1 | .0853749 | .1957399 | .4157034 |
| 2 | .0141092 | .0392708 | .1456024 |
| 3 | .0023283 | .0069822 | .0417406 |
| 4 | .0003843 | .0011810 | .0102595 |
| 5 | .0000634 | .0001963 | .0022383 |
| 6 | .0000105 | .0000324 | .0004459 |
| 7 | .0000017 | .0000054 | .0000830 |
| 8 | .0000003 | .0000009 | .0000148 |
| 9 | 0 | 0 | .0000025 |
| 10 | | 0 | .0000004 |

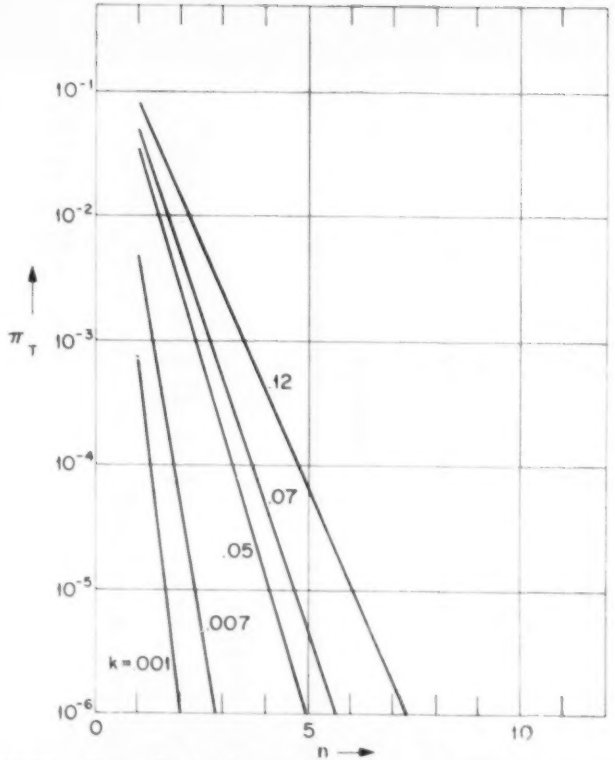


FIG. 7. The purification factor $\pi_T$ after the $n^{th}$ pass with indicated $k$ values as a parameter. The case for $l/L = 0.5$.

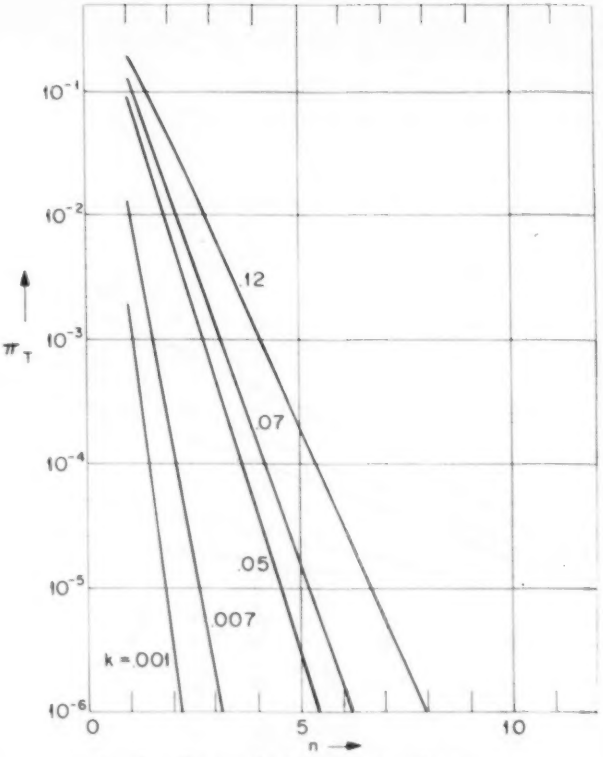


FIG. 8. The case for $l/L = 0.25$. (Cf. Fig. 7.)

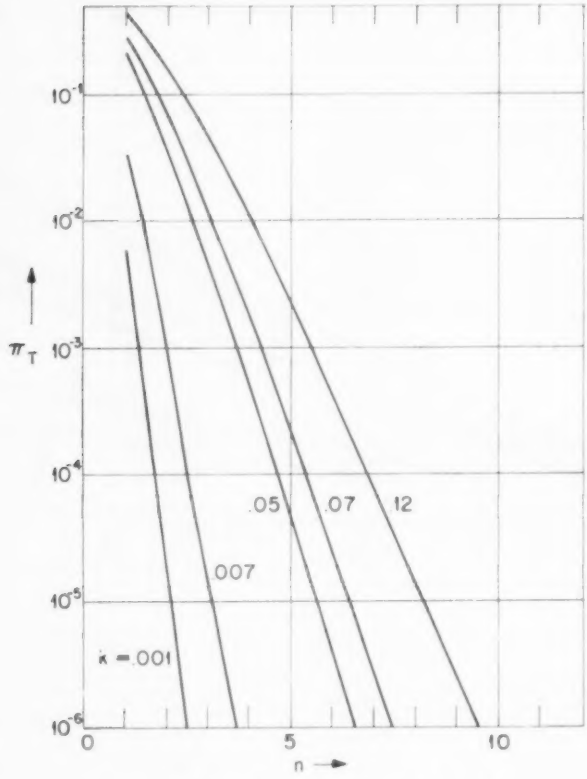


FIG. 9. The case for $l/L = 0.10$. (Cf. Fig. 7.)

## APPENDIX

The $Q_n(x)$ coefficients of Eq. 10*a* assume successively the values:

$$Q_1 = 1$$

$$Q_2 = 2 - e^{-k} + \left(\frac{k}{l}x\right)$$

$$Q_3 = (2 - e^{-k})^2 - ke^{-k} + (2 - e^{-k})\left(\frac{k}{l}x\right) + \left(\frac{k}{l}x\right)^2\Big/2!$$

$$Q_4 = (2 - e^{-k})^3 - 2ke^{-k}(2 - e^{-k}) \frac{k^2}{2} e^{-k} + [(2 - e^{-k})^2 - ke^{-k}]\frac{k}{l}x$$

$$+ (2 - e^{-k})\left(\frac{k}{l}x\right)^2 + \left(\frac{k}{l}x\right)^3\Big/3!$$

$$Q_0 = \left[(2 - e^{-k})^4 - 3ke^{-k}(2 - e^{-k})^2 - 2k^2e^{-k}(1 - e^{-k}) - \frac{k^3}{6}e^{-k}\right]$$
$$+ \left[(2 - e^{-k})^3 - 2ke^{-k}(2 - e^{-k}) - \frac{k^2}{2}e^{-k}\right]\frac{k}{l}x$$
$$+ \left[(2 - e^{-k})^2 - ke^{-k}\right]\left[\left(\frac{k}{l}x\right)^2/2!\right] + (2 - e^{-k})\left(\frac{k}{l}x\right)^3/3! + \left(\frac{k}{l}x\right)^4/4!$$

$$Q_0 = \left[(2 - e^{-k})^5 - 4ke^{-k}(2 - e^{-k})^3 - 6k^2e^{-k}(1 - e^{-k})^2\right.$$
$$\left. + \tfrac{3}{2}k^2e^{-2k} - \tfrac{3}{2}k^3e^{-k}(1 - 2e^{-k}) - \frac{k^4}{24}e^{-k}\right]$$
$$+ \left[(2 - e^{-k})^4 - 3ke^{-k}(2 - e^{-k})^2 - 2k^2e^{-k}(1 - e^{-k}) - \frac{k^3}{6}e^{-k}\right]\left(\frac{k}{l}x\right)$$
$$+ \left[(2 - e^{-k})^3 - 2ke^{-k}(2 - e^{-k}) - \frac{k^2}{2}e^{-k}\right]\left(\frac{k}{l}x\right)^2/2!$$
$$+ \left[(2 - e^{-k})^2 - ke^{-k}\right]\left(\frac{k}{l}x\right)^3/3! + (2 - e^{-k})\left(\frac{k}{l}x\right)^4/4! + \left(\frac{k}{l}x\right)^5/5! \quad \text{(A-1)}$$

whence the recursion formula (10*b*) is deduced.

Applying the boundary condition at $x = 0$, there obtains from Eqs. 7 and 9

$$\frac{C_n(0)}{C_0} = \frac{k}{l}\int_0^l \frac{C_{n-1}(x)dx}{C_0} = 1 - (1 - k)A_n$$

or

$$A_n = (1 - k)^{-1}\left(1 - \frac{C_n(0)}{C_0}\right) \quad \text{(A-2)}$$

In this manner Eqs. 10*c* and 10*d* follow with the first few coefficients identified as

$$A_1 = 1, \qquad A_2 = 2 - e^{-k}, \qquad A_3 = (2 - e^{-k})^2 - ke^{-k}$$

and

$$S_1 = e^{-k} - 1, \qquad S_2 = e^{-k}(k + 1) - 1, \qquad S_3 = e^{-k}(k^2 + 2k + 2) - 2.$$

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(1) L. GOLD, *J. Phys. Soc. Japan*, Vol. 14, p. 386 (1959).
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(3) W. G. PFANN, *Trans. A.I.M.M.E.*, Vol. 194, p. 747 (1952).

# LAMINAR SEPARATION OF FLOW AROUND SYMMETRICAL STRUTS AT ZERO ANGLE OF ATTACK

BY

PAUL K. CHANG [1]

ABSTRACT

This paper presents a recent rapid and accurate experimental technique for the separation of flow around a body surface and the comparison of experimental and analytical results on laminar separation point.

The experimental method used is called the "dust method." Dispersing fine dust in the separated region near the trailing edge, the separated region of any arbitrary shape of body could be determined.

The experiment shows that the dimensionless chordwise location of the laminar separation point of symmetrical and two-dimensional flow is constant for geometrically similar but different size strut models and is also independent of Reynolds numbers in the region of laminar flow.

The comparison of experimental and analytical data for the point of laminar separation of symmetrical and two-dimensional flow around a strut at zero angle of attack shows a good agreement.

## INTRODUCTION

A considerable number of research programs have been carried out on the subject of two-dimensional laminar flow separation, and highly detailed analyses as well as approximate calculations have had success in determining the laminar separation point. However, experiment has been difficult because of the time and expense required for such projects.

A recent simple and rapid experimental method, the so-called "dust method," has been introduced by W. Spangenberg, of the National Bureau of Standards, Washington, D. C.

This method consists of dispersing talcum dust in the separated region near the trailing edge of the model. The reverse flow in the separated region then carries the dust forward until the separation point is reached. The model is covered with an oil film so that the free dust particles will be entrapped in the surface oil film. The dust leaves a permanent record up to the farthest point of reverse flow, or the separation point. The simplicity of the method is of great advantage in determining the separated region of an arbitrary body shape, in view of the fact that analysis of flow separation around a three-dimensional body is difficult and inaccurate.

Since this dust method is comparatively new, an experimental evaluation in terms of classical experiments and analyses of known re-

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sults are required to establish its validity. Using a circular cylinder it was shown that the dust method yields sufficient accuracy in determining the laminar flow separation point. A more detailed description on this subject will be given under "test method."

The phenomenon of separation in general is caused by the simultaneous combination of an adverse pressure gradient and a loss in momentum of the inner boundary layer due to viscosity. The flow leaves the body surface at the point of separation which is mathematically defined as a point where $|\partial u/\partial y|_{wall} = 0$.[2] The result of this separation is an increase of form drag and decrease of lift.

Laminar separation is becoming of more interest with the development of low speed ground and water proximity vehicles, high Mach number missiles and its contribution to transition into turbulent flow (1).[3]

Using two groups of symmetrical and geometrically similar struts but with different chord lengths at zero angle of attack, it is shown by the dust method that the ratio of distance to the points of laminar separation with respect to the chord length remains constant for all models of each group and these points are independent of Reynolds number, and that the point of laminar separation depends only on the geometrical configuration.

A comparison of the locations of the laminar flow separation points for symmetrical struts at zero angle of attack, as predicted by the existing approximate methods with the results obtained by means of the dust method, shows good agreement.

## EXPERIMENTS

### 1. *Test Facilities and Apparatus*

The test facilities and apparatus used for laminar separation experiments are as follows:

(*a*) Low turbulent wind tunnel (Fig. 1)
(*b*) Dust dispenser (Fig. 2)
(*c*) Cylindrical model (Fig. 3)
(*d*) Symmetrical struts (Figs. 4 and 5).

The wind tunnel is of the low turbulence Eiffel (2) type, using a rectangular test section 2 ft. wide, 4 ft. high, and 14 ft. long. The intake air of the tunnel is supplied by a room (67 ft. × 135 ft. × 45 ft.), which is in effect a settling reservoir.

The dust dispenser consists of a metal container filled with talcum dust. Air pressure at 35 psi. was used in order to distribute the dust in the vicinity of the trailing edge of the model through the tubing shown in Fig. 2.

[2] Appendix B defines the notation used in this paper.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

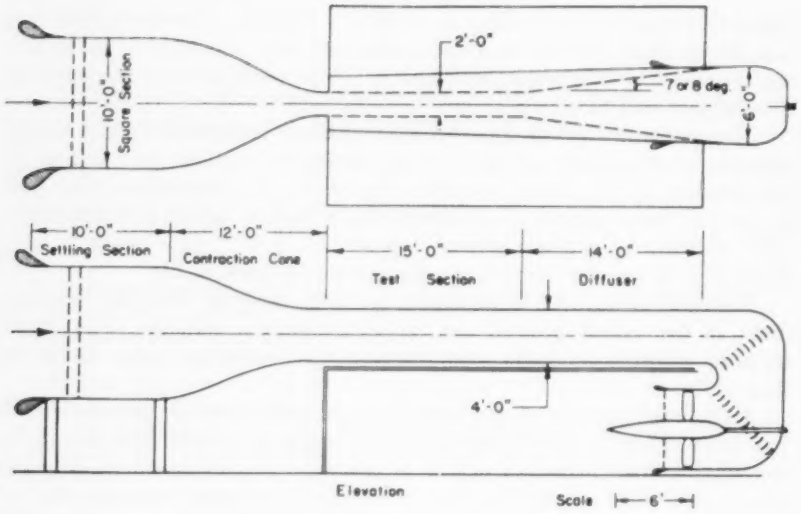


FIG. 1. Low turbulence wind tunnel.

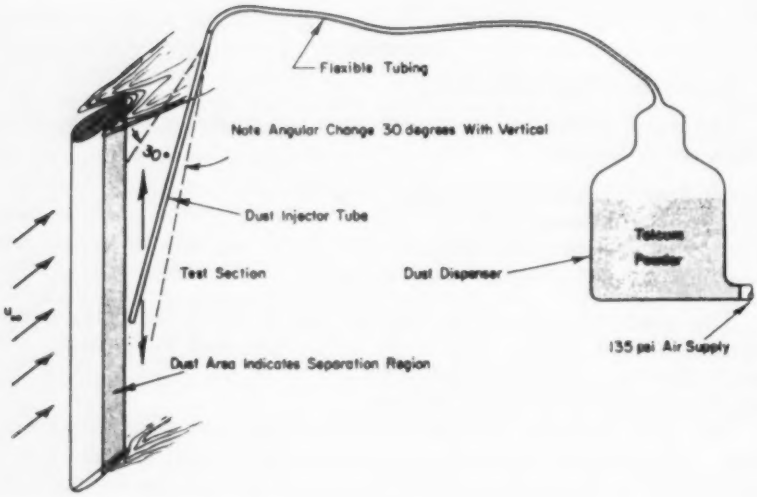


FIG. 2. Method of introducing dust into the separated region.

The experimental models consist of three basic shapes: a cylinder (Fig. 3) and thick and thin symmetrical struts (Figs. 4 and 5, respec-

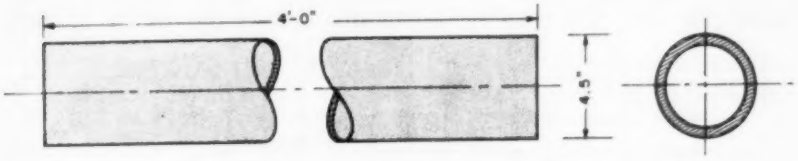


FIG. 3. Cylindrical model.

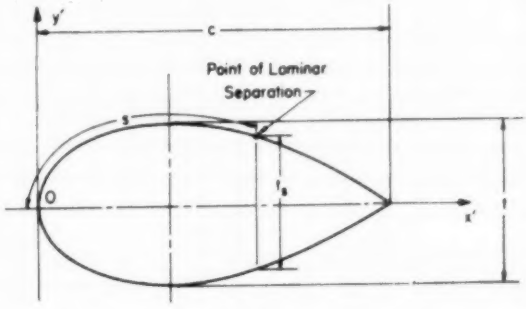


FIG. 4. Group I struts.

Nose radii = $0.57532\,\frac{t^2}{c}$, $\frac{t}{c} = 0.5$

$a = 0.4361302$

$b = 0.3083906$

$c$ = Chord

$t$ = Maximum thickness

$$0 \le \frac{x'}{c} < 0.436, \qquad y' = \frac{t}{2}\left\{1 - \left[\frac{1}{a}\left(a - \frac{x'}{c}\right)^2\right]\right\}^{1/2}$$

$$0.436 \le \frac{x'}{c} \le 0.872 \qquad y' = \frac{t}{2}\left\{\left\{1 - \left[\frac{1}{2}\left\{\frac{1}{a}\left(\frac{x'}{c} - a\right)\right\}^2\right]\right\}\right\}$$

$$0.872 \le \frac{x'}{c} \le 1.0 \qquad y' = \frac{t}{4}\left\{\left[\frac{1}{b}\left(b + 1 - \frac{x'}{c}\right)\right]^2 - 1\right\}^{1/2}$$

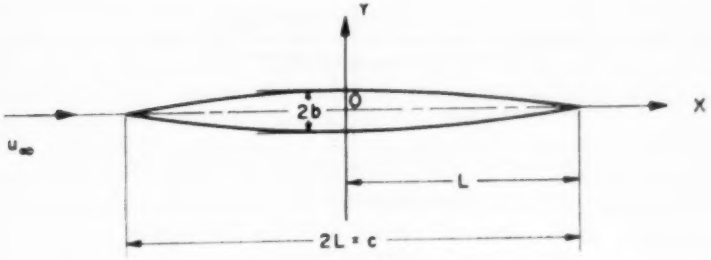


FIG. 5. Group II struts.

$$Y(X) = \pm b\left[1 - \left(\frac{X}{L}\right)^2\right]$$

or $$\eta = \pm b'(1 - \xi^2)$$

where $$\eta = \frac{Y}{L}, \quad \xi = \frac{X}{L} \quad \text{and} \quad b' = \frac{b}{L}$$

tively). The thick struts, hereinafter called Group I struts, have a 50 per cent thickness ratio and a geometry that is governed by four different equations which are continuous over three intervals of the chord (Fig. 4). The velocity distribution around the surface of these

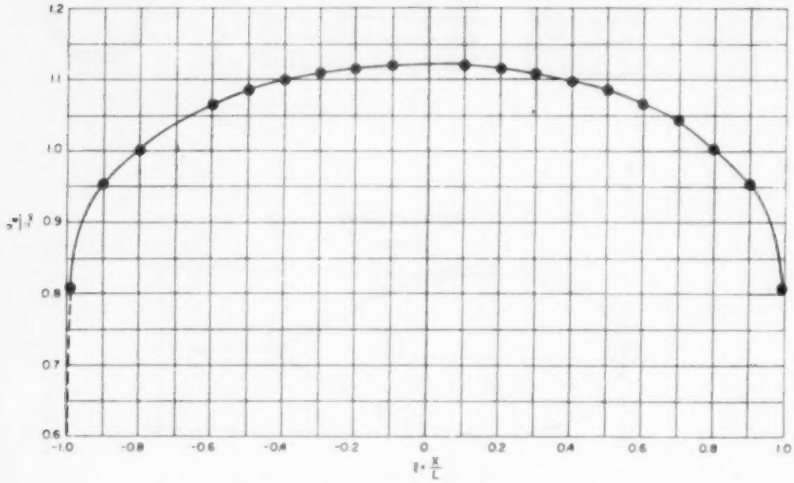


FIG. 6. Potential velocity distribution of Group II struts.

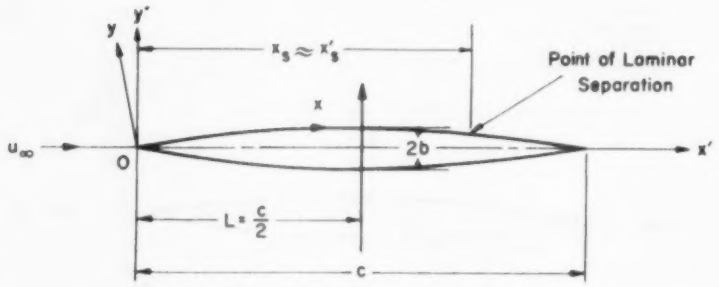


FIG. 7. Coordinate system of Group II struts.

| b, in. | c, in. | |
|---|---|---|
| 1.80 | 38.4 | |
| 0.90 | 19.2 | Length 2 ft. |
| 0.45 | 9.6 | |

models is not known. The thin struts (Group II) have a 9.42 per cent thickness ratio, and a known potential flow velocity distribution (Figs. 5, 6 and 7). Models belonging to both families of struts, but with different chord lengths, were tested.

2. *Test Method and Test Procedure*

The dust method has been used with good results by Smith and Murphy (3) of the Douglas Aircraft Co. for the study of separation on bodies of revolution.

To prepare the model for testing, a thin coat of SAE 10 lubricating oil is applied to the model's surface. Dust is then injected into the wake of the model near the trailing edge. If separation occurs, the back flow will carry the dust particles forward to the point of separation, and then away to the free stream. Dust particles will attach to that portion of the oil film which is in the separation region, leaving a permanent record of the point of separation.

There are, however, limitations to this method. Small perturbations in the position of the point of separation will not be recorded, since the method records only the most forward position of the dust.

In order to evaluate the accuracy of this dust method, a smooth aluminum hollow cylinder 4.5 in. in OD and 4 ft. in length (Fig. 3) was mounted vertically inside the wind tunnel. Fine dust was then dispersed uniformly in the wake of the model. A separation angle of 82.5° from the point of stagnation was measured.

Hiemenz (4) and Kreith, (5), working separately, found that laminar separation occurred between 80° and 85° on a circular cylinder at 90° to flow. From this favorable comparison it is believed the dust method shows sufficient accuracy for the determination of flow separation.

The symmetrical struts were tested at zero angle of attack in the Reynolds number region of laminar flow. The models were mounted vertically inside the wind tunnel as shown in Fig. 2. Dust was then introduced uniformly into the separated region of flow, by moving the dust injector tube up and down through a hole in the tunnel observation window in the ceiling of the tunnel (Fig. 2). The finely granulated particles were then carried forward in the separated boundary layer up to the point of separation. The thin oil film on the surface of the struts entrapped the dust up to the point of separation leaving the line of separation clearly marked (Figs. 8 and 9).

The disturbing influence of the chordwise component of the jet velocity and the presence of dust in the boundary layer of the separated flow region were evaluated in the following manner: the angular change of the dust jet varies 30° to 60° from the vertical. This variation in angle causes a change in the chordwise component of the jet by 1:1.73. However, the variation in chordwise position of the line of separation is of the magnitude of 1 per cent. It is therefore concluded that the chordwise component of the velocity of the dust jet is negligible.

The influence of dust on the aerodynamic characteristics of the free jet has been studied by Chernov (6). The results indicate that dust has little effect when the mass concentration is below 1 lb/lb air, and

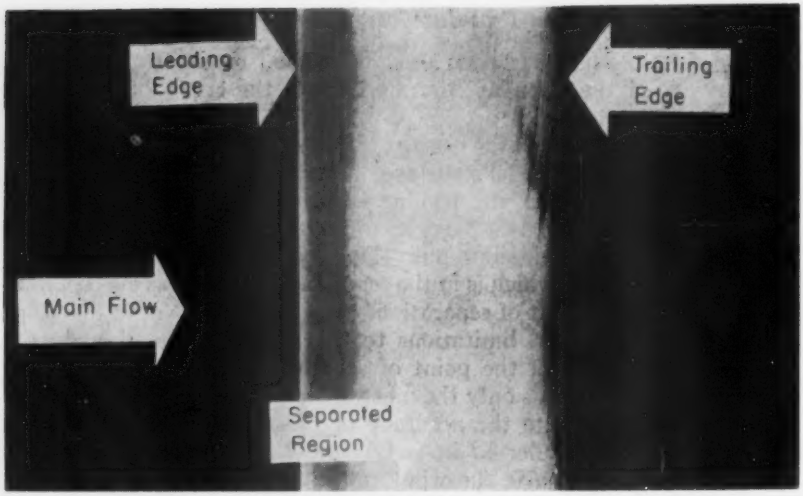


FIG. 8. Laminar separation of flow—Group I strut.

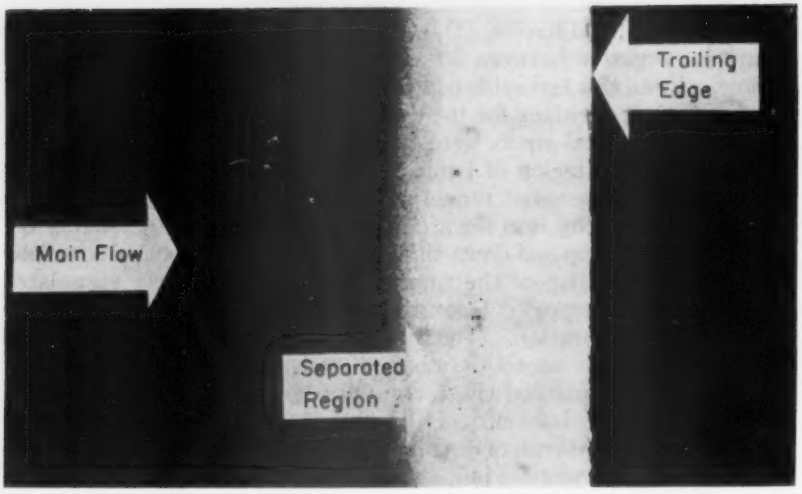


FIG. 9. Laminar separation of flow—Group II strut.

air jet velocities remain below 97 ft./sec. The mass concentrations of dust and velocities in this experiment are smaller than the values cited; therefore, the effect of the dust particles is assumed to be negligible.

*3. Test Results*

The measurements of laminar separation points of the symmetrical struts are shown in Tables I and II and also in Fig. 10.

TABLE I.—*Experimental Data for Laminar Separation Point.*
Group I Struts ($Re_c = 1.6 \times 10^4$ to $2.6 \times 10^5$)

| $c$, in. | $s$, in. | $\frac{s}{c}$ | $\frac{x}{c}$ | $\frac{l_s}{c}$ |
|---|---|---|---|---|
| 10 | 5.15 | 0.515 | 0.365 | 0.498 |
| 8 | 4.09 | 0.511 | 0.362 | 0.475 |
| 6 | 3.08 | 0.5133 | 0.385 | 0.483 |
| 4 | 2.04 | 0.510 | 0.375 | 0.460 |
| 2 | 1.06 | 0.530 | 0.375 | 0.475 |

TABLE II.—*Experimental Data for Laminar Separation Point.*
Group II Struts ($Re_c = 1.6 \times 10^4$ to $1.02 \times 10^5$)

| $c$, in. | $x_s$, in. | $\frac{x_s}{c}$ | $\frac{x_s}{c}$, average |
|---|---|---|---|
| 9.6 | 7.5 | 0.78 | 0.777 |
| 19.2 | 14.75 | 0.77 | |

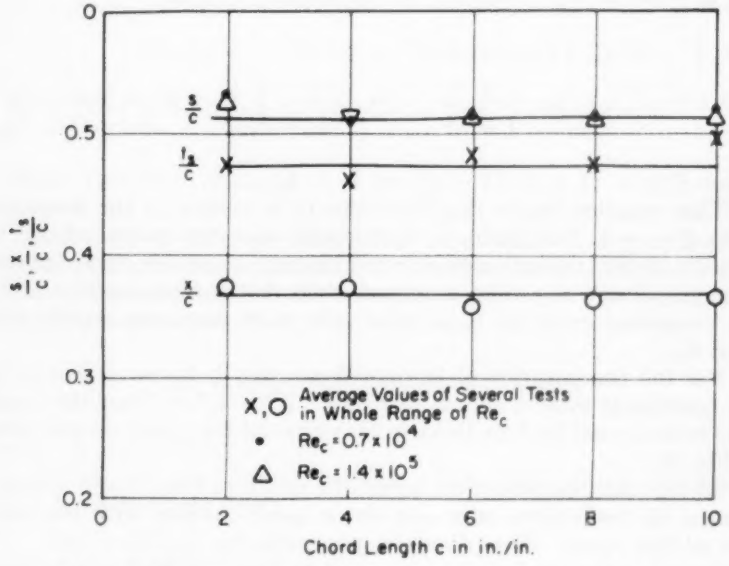


FIG. 10. Measurements of laminar separation points—Group I struts.

Several tests at a given tunnel velocity were conducted on each model. Each model was tested at several velocities. The positions of the separation points $s/c$ and $x_s/c$, shown in Tables I and II are averages of all the tests on each strut. It was observed that throughout the range of test Reynolds numbers $s/c$ and $x_s/c$ for each strut deviated insignificantly from the average. Therefore, the dimensional separa-

tion point can be regarded as independent of Reynolds number. The dispersion of the non-dimensional separation point at the test Reynolds numbers is shown in Fig. 10. Further, the tables show that the non-dimensional separation point is essentially constant for a given family of models.

## ANALYSIS

In order to compare the experimental and analytical results, three approximate analytical methods are applied for the determination of the laminar separation point on the thin symmetrical biconvex Group II struts.

The geometry of the Group II model is expressed by:

$$\eta = \pm b'(1 - \xi^2)$$

where $\eta = Y/L$, $\xi = X/L$ and $b' = b/L$.

The potential flow velocity distribution about this model configuration is given by Van Dyke (7) as follows:

$$u' = 1 + (2/\pi)b'\{2 - \xi \ln F(\xi)\} + (b')^2\left[\frac{3}{\pi^2}\{2 - \xi \cdot F(\xi)\}^2 - \left(\frac{1}{\pi^2}\ln^2 F(\xi) - (1 - \xi^2)\right)\right]$$

where $F(\xi) = (1 + \xi)/(1 - \xi)$ and $u' = u_e/u_\infty$.

This equation shows that the velocity is infinite at the stagnation point $\xi = -1$, but finite, in fact small, near this point. Since the velocity at the stagnation point must be zero, a suitable curve is drawn through point $\xi = -1$ in a manner such that it becomes identical to the computed curve of $u_e/u_\infty$ near this point but zero at this point (Fig. 6).

For the computation of laminar separation it is convenient to use the coordinate system shown in Fig. 7. This differs from the coordinate system used by Van Dyke in his potential flow analysis and shown in Fig. 5.

In the analysis presented herein the origin of the $x'$- and $y'$-axes is located at the leading edge and the $x'$-axis coincides with the center line of the strut. The $y'$-axis is perpendicular to the $x'$-axis. The distance $x$ is the arc length on the surface from the leading edge and $y$ is the normal distance to the surface. Since the radius of curvature of the surface of the Group II models is large, compared with chord length the arc length on the surface $x$ is approximated by the straight distance $x'$. Consequently, the arc length from the leading edge to the point of laminar separation, $x$, can be approximated by a straight distance $x_s'$.

A brief description of the three methods and their results are given in the following. A more detailed analysis is deferred to Appendix A.

1. *Stratford's Criterion* (**8, 9**)

A simple and accurate criterion for the prediction of laminar separation was formulated by Curle and Skan (**10**) by numerical comparison with the rigorous Görtler method (**11**). Computing the positions of separation for eleven different cases by Görtler's method and by Stratford's criterion given below, and comparing the results of both methods, it was shown that the average difference of the two methods is less than 3 per cent.

The Stratford formula may be expressed as

$$x^2 c_p \left(\frac{dc_p}{dx}\right)_s^2 = 0.0104$$

where $x$ is the distance measured from the initial point of adverse pressure gradient to the point of laminar separation. In the Group II models, $x$ is measured aft of the point of maximum potential velocity. Since $c_p = 1 - (u_e/u_{max})^2$, then

$$dc_p/dx = -2(u_e/u_{max}) \cdot \{d(u_e/u_{max})\}/dx.$$

Calculations show that laminar separation occurs at $x_s'/c = 0.8346$ which differs from the experimental mean by 6.5 per cent.

2. *Shvets' Criterion* (**12**)

The Shvets method of prediction is the result of a second-order approximation to the solution of a boundary-layer equation. This solution is then evaluated at the boundary condition of $(\partial u/\partial y)_{wall} = 0$.

Laminar separation occurs if the following relation is satisfied

$$4(du'/d\xi)(u')^{-6} \int_0^{\xi} \{u'(\xi)\}^5 \cdot d\xi = -1$$

where $u' = u_e/u_\infty$ and $\xi = x/L$.

Calculations show that $s/c = 0.77$. This differs from the experimental mean by 1 per cent.

3. *Thwaites' Criterion* (**13**)

Thwaites' method was modified by Curle and Skan (**10**) and its criterion for laminar separation is given by

$$m = -(\theta^2/\nu)(du_e/dx) = 0.090,$$

where $\theta$ is momentum thickness of boundary layer and may be computed by Truckenbrodt's method (**14**).

Calculations give laminar separation at $s/c = 0.7025$, which is 9.7 per cent from the experimental mean.

4. *Dust Method*

For the same models used in the above computation, the experimental results show $s/c = 0.77$ with a 1.02 per cent extreme variation from the mean. A comparison of the experimental results and the analytical predictions of the above-mentioned methods of calculation is shown in Fig. 11.

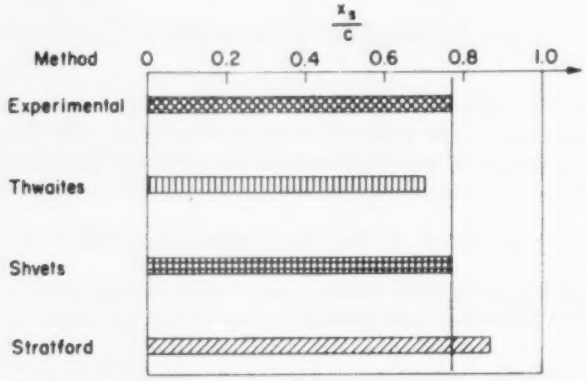


FIG. 11. Relative accuracy of analytical methods compared to experimental results.

The experimental results for Group II models gave an average $s/c = 0.777$ with a 1.02 per cent maximum deviation from the mean (Table II). A comparison with the analytical predictions of the several criteria given above is shown in Fig. 11.

## DISCUSSION

The measured position of the two-dimensional laminar separation point lies within the positions obtained from the analytical methods. The reason for this discrepancy among the analytical results may be explained by Schlichting's (4) analysis of the position of separation. Schlichting points out that laminar separation can be calculated with a certain degree of uncertainty particularly in the case when the point of separation is situated comparatively far behind the point of minimum pressure.

For Group II (thin) models, the position of minimum pressure of potential flow is located at mid-chord and the point of two-dimensional laminar separation is located considerably downstream. Hence, the analytical results can be expected to yield only approximations of the location of the laminar separation point.

## CONCLUSIONS

From experiments with the dust method and comparisons with analyses at zero angle of attack in two-dimensional flow, the following conclusions are made:

1. The dust method is a rapid and accurate method for the determination of the laminar flow separation point of bodies.
2. Experiments of symmetrical and geometrically similar struts show that the point of laminar separation is independent of Reynolds number and is dependent only on the geometric configuration.
3. Good agreement was obtained between the analytical results and the experimental data for the location of the laminar separation point.

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## APPENDIX A

1. *Stratford's Criterion* (**8, 9, 10**)

Stratford solved the boundary layer equations in the outer and inner layers.

In the outer boundary layer he showed that it is possible to consider separately the effects upon the velocity profile of a pressure gradient in the absence of viscosity and the effects of viscous forces upon the velocity profile in the absence of a pressure gradient.

In the inner sublayer there is balance between the pressure gradient and viscous forces. Hence, he concluded that there is a transition across the layer such that pressure forces are balanced by viscous forces at the wall, and by inertia forces at the outer edge of the boundary layer.

Stratford obtained mathematically an expression for the condition of separation covering all possible values of separation distance based upon Blasius' solution.

Curle and Skan (**10**) modified Stratford's criterion by comparing it with Görtler's (**11**) rigorous solution and found the following criterion yields better results:

$$x^2 c_p (dc_p/dx)^2 = 1.04 \cdot 10^{-2}$$

where $x$ is measured from the initial point of adverse pressure gradient (in our case $x$ is measured aft of the point of maximum potential velocity) and $c_p = (p - p_{\min})/\frac{1}{2}\rho u^2_{\max}$. Therefore,

$$dc_p/dx = -2\frac{u_e}{u_{\max}} \cdot \frac{d(u_e/u_{\max})}{dx}$$

and

$$x^2 c_p (dc_p/dx)^2 = 4\{1 - (u_e/u_{\max})^2\}\{\xi \cdot (du'/d\xi)\}^2 \left(\frac{u_e}{u_{\max}}\right)^2 \left(\frac{u_\infty}{u_{\max}}\right)^2.$$

For Group II models,

$$\frac{u_e}{u_{\max}} = \frac{u_e}{u_\infty}\frac{u_\infty}{u_{\max}} = \left[1 + \left(\frac{2}{\pi}\right)b'\{2 - \xi \ln F(\xi)\} + (b')^2\left\{\frac{3}{\pi^2}\{2 - \xi \cdot F(\xi)\}^2 - \frac{1}{\pi^2}\ln^2 F(\xi)\right\} - (1 - \xi^2)\right]\left[1 + \frac{4}{\pi}b' + (b')^2\left\{\frac{12}{\pi^2} - 1\right\}\right]^{-1}$$

and

$$\frac{du'}{d\xi} = -\frac{2}{\pi}b'\left\{\ln F(\xi) + \frac{2\xi}{1 - \xi^2}\right\} + 2(b')^2\left[-\frac{3}{\pi^2}\{2 - \xi \ln F(\xi)\}\left\{\ln F(\xi) + \frac{2\xi}{1 - \xi^2}\right\} - \frac{4}{\pi^2}\frac{\ln F(\xi)}{1 - \xi^2} + \xi\right]$$

where

$$\xi = x/L, \qquad F(\xi) = (1 + \xi)/(1 - \xi) \qquad \text{and} \qquad u' = u_e/u_\infty.$$

2. *Shvets' Criterion* (**12**)

The boundary layer equation with pressure gradient is

$$\frac{\partial^2 u}{\partial y^2} = u\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\int_0^y \frac{\partial u}{\partial x}\,dy - u_e\frac{du_e}{dx}$$

with the boundary conditions

$$u = 0 \quad \text{at} \quad y = 0$$
$$u = u_e \quad \text{at} \quad y = \delta.$$

Solving this equation to a second-order approximation

$$\frac{u}{u_e} = \frac{\delta^2}{24}\frac{du_e}{dx}(\varphi^4 - 12\varphi^2 + 11\varphi) - \frac{\delta}{24}\frac{d\delta}{dx}u_e(\varphi^4 - \varphi) + \varphi$$

where

$$\varphi = y/\delta.$$

Using

$$(\partial u/\partial \varphi)_{\varphi=1} = 0$$

$$\frac{3\delta^2}{8}\frac{du_e}{dx} + \frac{\delta}{8}u_e\frac{d\delta}{dx} = 1.$$

Assuming that when $x = 0$, $\delta = 0$ and solving for $\delta^2$

$$\delta^2 = \frac{16}{u_e^6}\int_0^x u_e^5 dx.$$

Evaluating the shearing stress at the wall,

$$\left.\frac{\partial u}{\partial y}\right|_{\text{wall}} = \frac{u_e}{\delta}\left[\frac{4}{\delta} + \delta\frac{du_e}{dx}\right] = \frac{u_e^4}{3}\left(\int_0^x u_e^5 dx\right)^{-1/2}\left[1 + 4\frac{\frac{du_e}{dx}}{u_e^6}\int_0^x u_e^5 dx\right].$$

But at the separation point, $(\partial u/\partial y)_{y=0} = 0$. This condition is satisfied when

$$4\frac{du_e}{dx}\Big/u_e^6\int_0^x u_e^5 dx = -1.$$

In non-dimensional form, this equation becomes

$$\{4(du'/d\xi)/(u')^6\}\cdot\int_0^\xi (u'(\xi))^5 d\xi = -1.$$

3. *Thwaites' Criterion* (**13**)

The Thwaites criterion modified by Curle and Skan (**10**) for laminar separation is

$$m = -\frac{du_e}{dx}\frac{\theta^2}{\nu} = 0.09.$$

The momentum thickness $\theta$ is computed using Truckenbrodt's (**14**) method.

$$\frac{\theta}{l} = \frac{\left[c_1^* + \left(\frac{c_f}{2}\right)^{1+n}\int_{x_1/l}^{x/l}(u_e/u_\infty)^{3+2n}(R/l)^{1+n}d(x/l)\right]^{1/(1+n)}}{(u_e/u_\infty)^3\cdot(R/l)}$$

where $l$ is a characteristic length. Here $l$ is replaced by $L$. Since the flow is fully laminar from the initial position at $x_1 = 0$, $c_1^* = 0$, and $n = 1$.

For two-dimensional flow, $R$ disappears.

Furthermore, for laminar flow,

$$\frac{c_f}{2} = \sqrt{0.441}\left(\frac{\nu}{u_\infty x}\right)^{1/2}.$$

Hence

$$\theta^2 = \frac{0.441\cdot\nu\cdot x}{u_\infty(u_e/u_\infty)^6}\cdot\int_0^{x/L}(u_e/u_\infty)^5 d\left(\frac{x}{L}\right).$$

Therefore, the criterion in dimensionless form is

$$m = -0.441\frac{du'}{d\xi}(u')^{-6}\cdot\xi\int_0^\xi (u')^5 d\xi.$$

## APPENDIX B

*Notation*

$b$ = one half of the thickness
$b'$ = $b/L$
$c$ = chord length of strut
$c_f$ = coefficient of friction $c_f = \tau/\frac{1}{2}\rho u_e^2$
$L$ = one half of the chord
$m$ = $-(\theta^2/\nu)(du_e/dx)$
$n$ = parameter, velocity profile: $\frac{u}{u_e} = \left(\frac{y}{\delta}\right)^n$
$p$ = static pressure
$R$ = radius
$Re_c$ = Reynolds number based upon chord length and undisturbed free stream velocity
$s$ = arc length between leading edge and separation point
$t$ = maximum thickness of Group I strut
$t_s$ = thickness at laminar separation point of Group I strut
$u$ = streamwise velocity within the boundary layer
$u'$ = $u_e/u_\infty$
$u_e$ = streamwise velocity at the edge of boundary layer
$u_\infty$ = undisturbed flow velocity
$X$ = horizontal coordinate of Group II strut (origin is at the center of strut)
$x$ = arc length on the curved surface of strut from leading edge
$x'$ = horizontal coordinate of strut (origin is at the leading edge)
$x_1$ = value of $x$ where integration begins
$x_s$ = distance measured from leading edge to separation point
$Y$ = perpendicular axis to $X$
$y$ = perpendicular axis to $x$
$y'$ = perpendicular axis to $x'$
$\xi$ = $X/L$ or $\xi = x/L$
$\eta$ = $Y/L$
$F(\xi)$ = $(1 + \xi)/(1 - \xi)$
$\delta$ = boundary layer thickness
$\theta$ = boundary layer momentum thickness
$\rho$ = density
$\tau$ = shear stress
$\varphi$ = $y/\delta$

## READER OPINION

### DELAY LINE SECONDARY RESPONSES

D. C. Cooper [1]

I read with considerable interest the paper entitled, "Delay-Line Secondary Responses in AM and FM Sweep Integrators," by H. Urkowitz, published in the January 1960 issue of your JOURNAL.

The author of the article mentions in concluding that no quantitative results of experiments with AM sweep integrators were available to him, and it is in this connection that I feel that I can offer some information which may be of interest.

In the course of some work on sweep integration some observations were made on the effect of the secondary responses of a badly adjusted mercury delay line. The delay of the line was 750 $\mu$ sec. and due to poor alignment rather large secondary responses occurred in the neighborhood of integral multiples of the nominal delay. There was also a direct feed through secondary

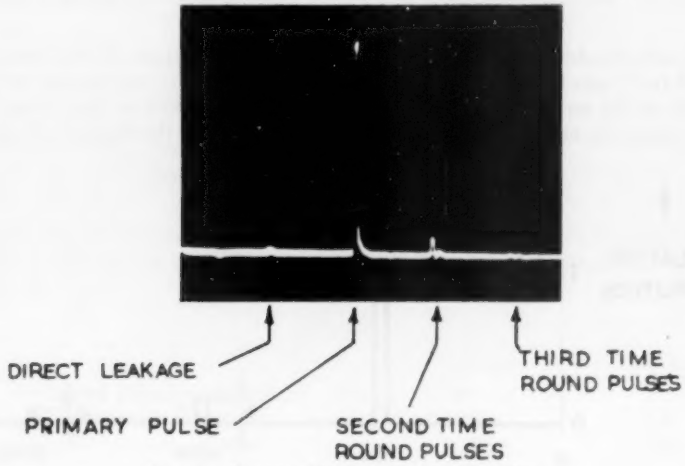


FIG. 1. Output of badly adjusted delay line.

response and Fig. 1 shows a photograph of the responses seen when a train of 2 $\mu$ sec. pulses with repetition time greater than three times the line delay was applied to the system. This photograph corresponds to Fig. 2 of the paper by Urkowitz.

The effect of the secondaries occurring just before each integral multiple of the delay time was of particular interest, since when the repetition period

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of the input pulse train corresponded to the delay of the line and an integrating loop was made, these secondaries produced large responses close to the desired response. A photograph of such a set of responses is shown in Fig. 2.

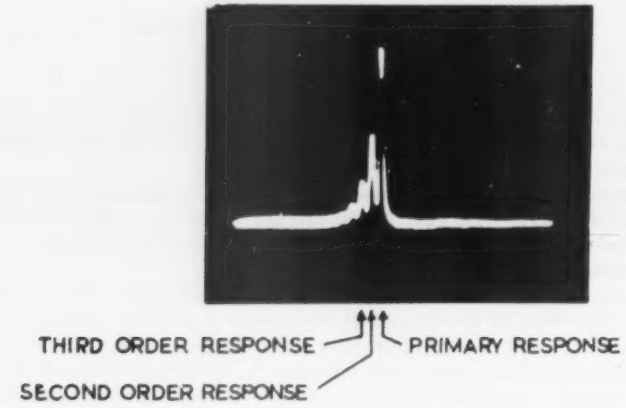


FIG. 2. Secondary responses in an integrator using the badly adjusted delay line.

In order to obtain a simple estimate of what the amplitude of these responses should be it was found necessary to neglect the effects of the second, or later, of each of the secondary pairs shown in Fig. 1. Secondaries with more than three times the nominal delay were not considered and the direct leakage was

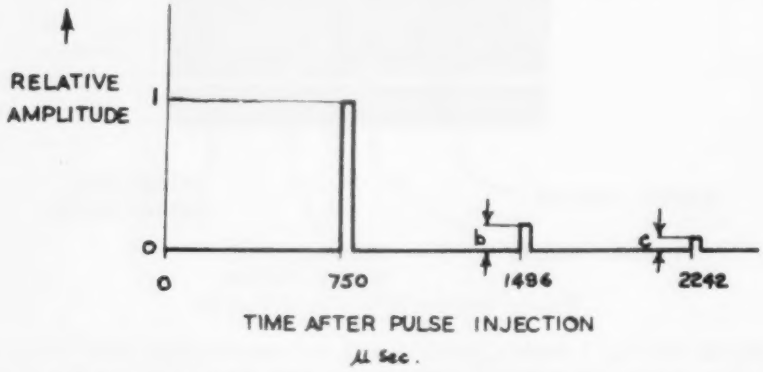


FIG. 3. Simplified delay line output assumed for the calculation of build-up.

assumed to be allowed for in the definition of loop gain. The simplified set of secondaries is sketched in Fig. 3.

The expressions obtained for the magnitudes of the second- and third-order responses were as follows,

$$\text{Second-order response} = \frac{b}{(1-k)^2}$$

$$\text{Third-order response} = \frac{c}{(1-k)^2} + \frac{b^2}{(1-k)^3}$$

where $k$ = loop voltage gain.

It is interesting to note that the magnitude of the second-order response agrees with that obtained by Urkowitz but that the third-order response has an additional contribution which arises from the fact that the spacing between the secondaries we are considering is precisely equal to the spacing between the required output pulse and the first of these secondaries. It would therefore seem erroneous to take the expression for the build-up ratio derived by Urkowitz as applicable to all cases, and since some regularity in the arrangement of secondaries is probable, greater build-up-ratios are quite likely to occur.

An experimental check on the above expressions was made and the important results are outlined in Tables I and II.

TABLE I.—*Delay Line Responses (Relative Amplitude).*

| Times of Occurrence, $\mu$ sec. | Relative Amplitude |
|---|---|
| 0 | 0.05 |
| 750 | 1.00 |
| 1496 | 0.15 |
| 1500 | 0.03 |
| 2242 | 0.015 |
| 2246 | 0.01 |

We see that for our simplified set of secondaries we take $b = 0.15$ and $c = .015$. With a loop gain $k$ of 0.75, relative amplitudes were obtained, as shown in Table II.

TABLE II

| | Measured | Calculated |
|---|---|---|
| Wanted Primary Resultant | 1 | 1 |
| Second Order Resultant | 0.6 | 0.6 |
| Third Order Resultant | 0.45 | 0.42 |

These results show reasonable agreement between calculation and experiment but this was not so with higher loop gains, presumably because of the assumptions made in order to obtain simple theoretical expressions.

So much for the effect of secondary responses. What about methods of reducing their effect?

In this respect Urkowitz has described the FM transmission scheme which I think is most elegant. However I do not share his pessimism about improving the AM sweep integrator without using high quality delay lines. Perhaps I may be allowed to expound my ideas.

I shall consider that by AM we mean an envelope modulated transmission in the delay line followed by an envelope detector. This is the type of system to which my previous remarks apply.

In such a system we may regard the transmission of a positive going pulse as a burst of carrier oscillation added in phase to the constant residual carrier. The arrival of the residual carrier and the carrier pulse at the output transducer of the delay line does not necessarily mean that a positive going output envelope will result since the relative phase of the two components is of importance.

In the case of the required or primary signal each component has traversed the same path and the input phase relationship is maintained to produce a positive going envelope. However pulses of carrier arriving via longer or shorter paths will have phases which differ from that of the residual carrier and the difference will be a function of the carrier frequency, velocity of propagation, and path length difference. It is therefore quite possible for the secondary responses to produce a negative going output envelope or even an output with no pulse form at all.

In the observations on the badly adjusted delay line it was found that the polarity of the detected output secondaries of the delay line could be changed by adjustment (very slight) of the carrier frequency, and with synchronous input the integrated output of the system could be changed from the form of Fig. 2 to that shown in Fig. 4. Here we see that the even order re-

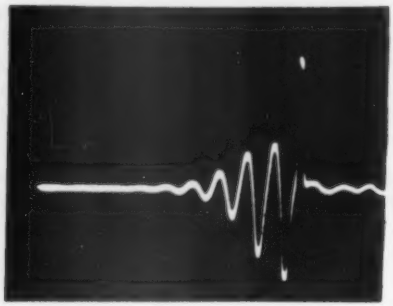

FIG. 4. Response of integrator showing the polarity reversal of even-order pulses.

sponses have changed polarity. Between the states of Figs. 2 and 4 a carrier frequency could be found which practically eliminated the unwanted responses, but this was undoubtedly due to a fortuitous set of circumstances.

It cannot be hoped that conditions will normally permit all the secondary responses to be eliminated by careful adjustment of carrier frequency but some reduction of amplitude will result if the carrier frequency is made to wander about in a way that is analogous to the changing centre frequency used in the FM system described by Urkowitz.

I regret that I have not found it necessary to try this method for improving the AM system but I feel sure that something can be done along the lines I have suggested.

## BOOK REVIEWS

DIRECT CONVERSION OF HEAT TO ELECTRICITY, edited by Joseph Kaye and John A. Welsh. v. p., illustrations, 6 ×9 in. New York, John Wiley & Sons, Inc., 1960. Price, $8.75.

The direct conversion of heat into electricity is a field which is rapidly growing in importance. Such devices promise lightweight, high-temperature, noiseless operation with few or no moving parts. One rather immediate application is the production of electricity for space vehicles, a task for which certain of these converters appear well qualified. Another potential application is operation thermally in series with conventional steam turbines in order to obtain more electricity per pound of fossil or nuclear fuel.

This book is a collection of 23 papers, about half of which have previously appeared in various technical journals. There are 11 papers on thermionic converters, 2 on magnetohydrodynamic converters, 9 on semiconductor converters, and one on fuel cells. The inclusion of a fuel cell paper seems somewhat extraneous since a fuel cell is not, by itself, a converter of heat to electricity.

This book is not a textbook and makes no attempt to give a complete, systematic presentation of each area of interest. Indeed, for many of these areas such a presentation is an impossibility since the devices are not completely understood. For instance, one paper concerning cesium-filled thermionic converters attributes the measured emf to the thermoelectric power of the plasma, while another paper, whose experiments involve the same range of cesium pressure, attribute the output voltage to quite a different source. Another symptom of the newness of the field is the fact that the thermionic converter is given at least six different names by various authors, including "thermoelectron engine," "cesium plasma diode," and "plasma thermocouple."

The papers are well-written and are highly recommended, both to workers in the field as a convenient reference, and to anyone interested in a survey of direct heat-to-electricity conversion as of mid-1959.

JOHN M. HOUSTON
*General Electric Research Lab.*

SOLID STATE PHYSICS. VOLUME 6. METHODS OF EXPERIMENTAL PHYSICS, edited by K. Lark-Horowitz and Vivian A. Johnson. Part A: Preparation, Structure, Mechanical and Thermal Properties, 466 pages, illustrations, 6 × 9 in. New York, Academic Press, 1959. Price, $11.80. Part B: Electrical, Magnetic, and Optical Properties, 416 pages, illustrations, 6 × 9 in. New York, Academic Press, 1959. Price, $11.00.

The tremendous increase of basic research in solid state physics during the past decade is being reflected by the numerous books on this subject. Almost without exception, they are treatises with strong theoretical leanings or purely theoretical in character. It is not surprising, however, that the fast development in any particular field of physics leads to new experimental techniques besides the classical ones which may have been altered and adjusted to suit special requirements of the new activity. Volume 6 of Marton's series "Methods of Experimental Physics" is the first attempt to present a collection of all experimental methods and techniques in solid state physics that are applied today.

The subject matter is organized in two parts each consisting of a book of over 400 pages. Part A contains the methods for the measurement of mechanical properties preceded by a chapter on specimen preparation and one on crystal structure determination. Part B deals with electrical, magnetic and optical measurements.

It is refreshing to read this volume because something of the challenge that is experienced by each research worker in this complex and fascinating field has been preserved in the articles. Undoubtedly, this must be due to the fact that more than 60 authors have been contributing, all of which have been active in their respective research fields. Alone, the

chapter on electrical properties lists thirteen authors. At the same time, it must have been an enormous task for the editors to achieve a balanced account. To some extent, frequent oral communication between editors and authors has been possible; for instance, nearly one third of the contributors to Part A belonged to the same university department.

During the past few years one has become increasingly aware of the history of the sample to be examined. A chapter on the "Preparation and Purification of Materials," constituting more than one third of Part A, was therefore most welcome. If it would not have been for the necessary space limitation, one would have wished that this part was even more extensive. This chapter, however, appears to be the most unbalanced of the volume regarding both the level of discussion and the choice of topics. One wonders for whom the article "Preparation of Laboratory Glass Samples" was intended. Since the overwhelming number of solids is crystalline, it was appropriate to present a survey of diffraction methods for determining the crystal structure. This is a very clear and precise account, mainly describing the well known x-ray and electron diffraction techniques. Unfortunately, neutron diffraction is described on only four pages out of seventy-six on diffraction. Here, it is felt, a more detailed account of this recent and least known method would have been in order.

The bulk of the two books is devoted to investigational methods concerning mechanical, electrical, magnetic and optical properties. A listing of these chapters is given since some unusual methods that have been developed rather recently have received detailed attention: Mechanical Properties, Thermal Properties, Solid State Studies under High Pressure, Electrical Properties (including High Frequency Effects in Semiconductors; Surface States), Galvanomagnetic and Thermomagnetic Effects, Magnetic Properties, Optical Properties (including Electronic Band Structure of Solids by X-ray Spectroscopy), Luminescence, Photoelectric Phenomena.

From the above listing, the fundamental approach that prevails throughout the volume becomes apparent. And here lies its main virtue. The books contain not just a collection of recipes for making certain measurements but introduce the reader to the underlying physics at a well chosen level. Much work has been done by the authors to present the subjects with clarity without losing depth. Most articles include research material published up to the beginning of 1958. Here and there one misses an improvement to a known method, or a technique which is used by rather small research groups, as for example, the Bitter pattern technique for making magnetic domains visible to the eye. But these omissions and some misspelled names in the numerous references are minor points that can easily be improved upon in the following edition. This volume is one of the few that will reach a large circle of readers as it had been intended for ". . . the advanced student, the teacher and the research scientist," in particular those of the latter group who wish to inform themselves about fields other than their present activities.

H. G. F. WILSDORF
*The Franklin Institute Laboratories*

FROZEN FREE RADICALS, by G. J. Minkoff. 148 pages, diagrams, 6 × 9 in. New York, Interscience Publishers, Inc., 1960. Price, $5.00.

The trapping of free radicals (and of other chemical species unstable under ordinary conditions) in inert matrices at low temperatures has been one of the more highly publicized areas of physical-chemical research in recent years. Much of the work has been carried out in connection with the National Bureau of Standards' three-year free-radical research program; and an alumnus of that program has now written what the publishers call "the first monograph on the subject by a single author."

It is probably best to consider the book as an annotated bibliography. From that point of view it will be useful to the researcher interested in finding out what has been done with regard to a given problem through approximately the end of 1958. Chapters 3 to 7 (Experimental Methods, Basic Principles and Requirements of Trapping, Studies of Individual Atoms, Diatomic and Triatomic Radicals, and Polyatomic Radicals Containing Carbon) give an adequate survey of a range of topics which extends from the design of Dewars for spectroscopy at liquid helium temperatures through the statistical mechanics of trapping to the experimental evidence for

the NH radical and the polymerization of acrylonitrile.

Unfortunately, the book does not qualify as a text on frozen free radicals. It has all the earmarks of a hastily assembled first draft, never revised by either author or editor. The writing is unbelievably sloppy. There is one truly inspired sentence: "Paramagnetic resonance, in collaboration with Professor Weissman, confirmed the presence of radicals in some rather elegant experiments." There are others whose grammar and syntax render unintelligible the description of an entire experiment. Again, a large number of technical terms and abbreviations (from atomic and molecular spectroscopy, electron spin resonance, chemical kinetics, etc.) are introduced without adequate explanation. It is twice pointed out that the initials ESR stand for electron spin resonance, but not once that the initials EPA refer to a mixture of 5 parts ether, 5 parts *iso*pentane, and 2 parts ethanol. Finally, there seems to have been very little in the way of a critical examination of the source materials. In one instance, a printer's error of a factor of $10^{10}$ in the original paper has been copied exactly, even though the mistake should have been obvious from the context: vibrational frequencies are of the order of $10^{14}$, and not $10^{4}$, $sec^{-1}$.

There is one point where the author really warms to his task: in a fine one-and-a-half page description of the manufacture and calibration of carbon resistor thermometers. For some readers, it may be worth the price of the book.

ALFRED BÜCHLER
*Arthur D. Little, Inc.*

INTRODUCTION TO SOLIDS, by L. V. Azaroff. 460 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $9.50.

There are already two good textbooks on solid state physics published in the last few years, and authored by Kittel and Dekker. These books, less theoretical and more up-to-date than Seitz's "Modern Theory of Solids," are intended for physics students at a senior or first-year graduate level and thus require a strong undergraduate background in physics. L. Azaroff's book is written on a more intermediate level and is mathematically less sophisticated. Such a book was needed especially by metallurgy students who will appreciate the coherent presentation by Azaroff of topics fundamental both in physics and in metallurgy.

The book is skillfully written and some of the topics are covered with a thoroughness and clarity lacking in the two books mentioned earlier. Among these topics let us mention the excellent treatments of crystallography and space groups, the transformations in crystals and their mechanical properties. The chapter on atomic packings is excellent, in particular the treatment of atomic radii, Pauling's rules and their applications. The quantum theory of the electronic structure of metals is necessarily sketchy, and so is the treatment of the magnetic properties. There is very little or nothing on ferromagnetic or on nuclear resonance.

Here, in brief, is the table of contents: Geometrical Crystallography; The Structure of Crystals; Atomic Packings in Crystals; Imperfections in Atomic Packings; Formation of Crystals; Transformations in Crystals; The Bonding of Atoms; Properties of Metals; Structure of Metals; Properties of Semiconductors; Structure of Semiconductors; Properties of Insulators; Structure of Insulators. There are also 4 appendices on Physical Constants, Conversion Units, Atomic Radii and Space Group Symbols.

All crystallographic and structural aspects of solids are unusually well presented. This will be a successful text for metallurgy students, and will be very useful for physics students.

HENRI AMAR
*The Franklin Institute Laboratories and Temple University*

SODIUM CHLORIDE, edited by Dale W. Kaufmann. ACS Monograph 145. 743 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $20.00.

This is the most comprehensive book yet to be published on salt, and it is the first time so much information on the subject has been assembled in one publication. The book covers a wide range of subjects related to salt—geology, technology, uses, physiology, etc. In fact, there seems to be no subject omitted. Anyone desiring information on most any aspect of salt would find something in this book about the subject. Many readers

will find information on salt that it would be difficult to find elsewhere. This is especially true of much of the data on physical properties.

The book is comprised of chapters written by a large number of contributors. There is a wide range of background and experience with salt of these different authors, with the result that many chapters are much more complete than others. The chapters on geology are especially good. The chapter on solar salt production could justifiy more thorough treatment than only U. S. practices where production is relatively small. Salt mining and rock salt milling are discussed very thoroughly and present practices are accurately described. The chapter on brine wells is mostly devoted to describing problems relating to methods of operation either abandoned or being abandoned. The most modern methods of constructing brine wells—by fracturing or padding—are dismissed with a statement that these methods are "too new" for evaluation. Actually, there is much known about the fracturing and padding of brine wells that could be reported. Similarly, in the chapter on brine purification, there is no discussion of the new processes whereby the recycling of sodium sulphate is used to reduce the calcium sulphate level of the brine. This method has been widely used in Europe for many years and, more recently, in the United States.

The chapter on vacuum pan salt suffers from the fact that the authors have had no recent experience in the salt industry. As a result, to a large extent, emphasis is on practices and technical problems which were abandoned or solved many years ago. For instance, there are several pages devoted to the problem of tube salting in evaporators. The reviewer has not observed this phenomenon in the past 25 years.

The most authoritative chapters in the book are written by authors who have had long experience with the subjects about which they are writing. For instance, the Editor, who has worked for many years with rock salt dissolving, has written the chapter on salt dissolvers, which is both complete (except for dissolving of solar salt) and accurate (except for the frequent use of the work "lixator" which cannot be found in any dictionary). The chapter on salt petrography leaves considerable to be desired. This latter chapter is far from complete, as the author only reported results of the inspection of ten different samples of salt in writing the chapter. The inspection of 50 or 100 samples would be required to give a clearer understanding of the subject. The chapters on nutrition and physiology are interesting.

The monograph illustrates the problems of writing a book by many authors. The advantage is that it is possible to obtain a coverage of the subject that would be difficult to obtain otherwise. However, the fact does remain that it is a time-consuming process, with the result that so much time has elapsed in the preparation of the book that the book, in parts, is technically obsolete at the time of publication. It is somewhat surprising that this should be so in a supposedly stable business like the salt industry; however, the book points out the fact that in recent years, knowledge had been gained about salt and that the Editor should now make plans for a revised edition, in order to report this progress.

H. W. DIAMOND
*Morton Salt Company*

## BOOK NOTES

SCIENCE SURVEY 1, edited by A. W. Haslett and John St. John. 359 pages, plates, diagrams, 5⅛ × 8¼ in. New York, The Macmillan Co., 1960. Price, $5.75.

Produced with the cooperation of the British Association for the Advancement of Science, this new series promises to fill a real need—an authoritative book on recent developments in science, written for the general reader. Volume 1 covers advances in seven major fields: "The Stuff of the Universe"; "Arranging Atoms for Industry"; "Cells, Viruses, and Heredity"; "Radiation and Fall-Out"; "Organisms and Their Behavior"; "The Earth"; and "Space Exploration." Twenty-six contributions, all by outstanding specialists, explain such new ideas as steady-state theories of the universe, protein synthesis, tropospheric fall-out, shrinkage of clothing fabrics; the Mohole project, and future rocket engines. If subsequent volumes measure up to this first one, man will have no excuse for not having at least a passing acquaintance with the latest developments in science.

VIBRATIONS FROM BLASTING ROCK, by L. Don Leet. 134 pages, illustrations, 5⅛ × 8¼ in. Cambridge, Harvard University Press, 1960. Price, $4.75.

The author has prepared a thoroughly practical volume on all phases of blasting. It will be useful not only to engineers in the explosives fields, but also to contractors, explosives manufacturers and insurance researchers. Many of the results of the author's investigations are published here for the first time. Concise tables give performances of various explosives, strengths of different rocks, air-wave frequencies and pressures, and amplitudes of ground motion at different frequencies. One of the most interesting features of the book is the quantity of excellent photographs and diagrams—seventy illustrations in a 134-page book.

INFORMATION PROCESSING. PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON INFORMATION PROCESSING, UNESCO, PARIS 15–20 JUNE, 1959. 520 pages, diagrams, 8½ × 11⅝ in. Paris, UNESCO; München, R. Oldenbourg; London, Butterworths; 1960. Price, $25.00. Available in U. S. A. from Columbia University Press.

The Congress, whose proceedings this volume represents, was attended by 2000 experts from 39 countries. Under the presidency of Dr. Howard H. Aiken of Harvard, the symposium covered recent developments and the current state of research throughout the world. Since automation is of extreme importance in all branches of industry and government, the papers and discussions herein published should find a wide audience in all countries. The international character of the volume is at once apparent—English, French, German, Spanish and Russian are used. Introductory speeches to each session are published in both English and French; abstracts of all papers are printed in all five languages, preceding the full paper, printed in the language in which it was delivered. Considering the tremendous amount of editorial work necessary for such a cosmopolitan undertaking, the publishers are to be congratulated for having the volume ready for distribution in less than a year.

The seven chapters of the book are: I. Methods of Digital Computing (12 technical papers); II. Common Symbolic Language for Computers (5 papers); III. Automatic Translation of Languages (5 papers); IV. Pattern Recognition and Machine Learning (17 papers); V. Logical Design of Computers (15 papers); VI. Computer Techniques of the Future (6 papers); and VII. Miscellaneous Topics. Within each chapter, there are also opening and closing remarks, and twelve "colloquia" and "symposia" are scattered throughout the text (such as one on switching algebra).

The most intriguing section for this reviewer

is the one on machine translation; here the Harvard Computation Laboratory program is outlined, the COMIT system is described, and methods for translating Russian and Japanese by machine are considered. Progress in this field could do much to further international understanding and to quicken scientific advances.

INFORMATION AND DECISION PROCESSES, edited by Robert E. Machol. 185 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $5.95.

The twelve papers presented in this volume were originally given at a Symposium on Information and Decision Processes, held at Purdue University in April 1959. In most cases the papers are exact transcriptions of the speakers' texts from the conferences. Of the twelve contributing authors, nine are professional mathematicians, and the other three (an economist, a philosopher, and an electrical engineer) are also well known as mathematicians. Each paper is essentially on the subject of information theory or decision theory. However, the print throughout the book is so appallingly microscopic one is immediately conscious, after glancing at the first page, of the thought —why bother!

PROCEEDINGS OF THE FIFTH CONFERENCE ON MAGNETISM AND MAGNETIC MATERIALS. 419 pages, illustrations, 8 × 10½ in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $10.00.

Issued as a Supplement to Vol. 31, No. 5 of the *Journal of Applied Physics*, these proceedings cover recent developments (through November 1959) in magnetism and magnetic materials in the fields of physics, chemistry, metallurgy, crystallography, and other areas. 170 short contributions are divided into fifteen topics, covering, for example, anisotropy, resonance, magnetic films, magnetic salts, etc. The contributions range from descriptions of simple test techniques to theoretical studies.

ELEMENTARY INTRODUCTION TO NUCLEAR REACTOR PHYSICS, by S. E. Liverhant. 447 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $9.75.

Intended to serve as a textbook for an undergraduate course in nuclear reactor physics, this work will be welcomed by the several universities that are just beginning to offer degrees in nuclear science and engineering. Calculus and a knowledge of elementary differential equations will satisfy the math requirements for the course. After three introductory chapters on the fundamentals of nuclear physics, eleven more chapters deal with subjects specifically related to reactors, such as nuclear fission, thermal neutrons, the nuclear chain reaction, neutron diffusion, the critical equation, etc. Worked-out examples add to the effectiveness of the presentation.

THE PHYSICAL UNIVERSE, by Konrad Krauskopf and Arthur Beiser. 536 pages, illustrations, 6½ × 9½ in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $6.50.

A new text for a two-semester course in the essential elements of physical science, this work could be used either in the advanced placement courses offered in some secondary schools, or in college. The elements of physics, chemistry, astronomy and geology are covered in twenty-four chapters, which are well illustrated with photographs and diagrams. Each chapter lists a few well chosen items for "further reading." The book is quite up-to-date (it contains the Russian photograph of the far side of the moon). A separate study guide is available, to aid the student in mastering the subject matter.

INTRODUCTION TO LAPLACE TRANSFORMS FOR RADIO AND ELECTRONIC ENGINEERS, by W. D. Day. 183 pages, diagrams, 5½ × 8½ in. New York and London, Interscience Publishers, Inc., 1960. Price, $5.50.

This text was prepared specifically for the radio and electronics engineer whose college curriculum did not include a study of Laplace transforms and who now finds himself in real need of a workable knowledge of this subject. It is a practical approach, with many of the usual formal mathematical proofs omitted. The first five chapters will give the reader sufficient background to acquire some facility in the use of Laplace transforms; the remaining five chapters are not so detailed, since the author feels these chapters are introductions to more advanced treatments. The book is designed for home study.

ELEMENTARY THEORETICAL FLUID MECHANICS, by Karl Brenkert, Jr. 348 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $7.50.

This new text covers the basic topics of fluid mechanics, with emphasis on the derivation and application of the fundamental equations. A knowledge of the calculus is a prerequisite, since it is used in many of the proofs. The eight chapters deal with: (1) definitions and concepts; (2) fluid statics; (3) conservation of matter and the Euler equation; (4) momentum and angular momentum; (5) conservation of energy; (6) friction; (7) dimensional analysis and model study; and (8) potential flow. Problems are given at the end of each chapter and one of the appendices consists of a list of references for further study.

RUNNERLESS MOLDING, by Ernest P. Moslo. 162 pages, illustrations, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1960. Price, $4.95.

A concise survey of the theory, design, applications and economics of runnerless molding is provided in this small text. It describes in detail the various techniques of injection molding accomplished without the removal of a sprue and runner system on each injection molding shot. The author has included discussions of such recent developments as precompression and sequential molding. Many explanatory drawings show the actual construction of nozzles used for runnerless molding, minifolds and mold equipment. The book is primarily directed to the plastics and related industries by demonstrating the ways in which runnerless molding may be employed to eliminate sprue scraps, cut labor costs, minimize rejects and greatly speed production.

POLYSTYRENE, by William C. Teach and George C. Kiessling. 176 pages, illustrations, 5 × 7 in. New York, Reinhold Publishing Corp., 1960. Price, $5.00.

This book gives a simplified yet technically accurate introduction to the chemistry, properties, manufacture and uses of polystyrene. The greater emphasis throughout the text is on the applications of polystyrene materials by injection molding, by extrusion, by vacuum forming, and as a foam. The latest information has been included on the uses of polystyrene in appliances, electronics, building, packaging, housewares, toys and novelties, shoe heels, transportation, and furniture. An entire chapter is devoted to the manufacture, fabricating, techniques and various uses of expandable polystyrene. A large amount of the material will be of interest to research workers, such as a new approach to the evaluation of the effects of polymerization temperature on molecular weight and new information on the copolymerization of styrene and alphamethylstyrene. However, it is directed primarily to molders, fabricators and end-users of polystyrene.

INTERNATIONAL MISSILE AND SPACECRAFT GUIDE, by Frederick I. Ordway III and Ronald C. Wakeford. v.p., illustrations, 8½ × 11 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $25.00.

In their preface, the authors state that "The Guide is *not* a primer on rocketry; it is, rather a handbook on specific missile systems." The authors make good their word, for the *Guide* covers four types of modern missiles (surface-to-surface, surface-to-air, air-to-surface, and air-to-air), rocket airplanes, manned missiles, spacecraft, research rockets, drones, and space-research vehicles in eleven countries. It also describes the history of rocket development, including an invaluable chronological table of missile progress, dating from 1200 to 1959. Many rare photographs add to the presentation. Data are included on launchings, warheads, operational uses, servicing, field maintenance, test programs, and technical specifications. Arranged by countries, the information is readily accessible. The *Guide* is an extensive and accurate survey; as such, it is a real contribution to space technology.

HANDBOOK OF SUPERSONIC AERODYNAMICS. VOLUME 6, SECTION 18, SHOCK TUBES, by I. I. Glass and J. Gordon Hall. 604 pages, diagrams, 8 × 10½ in. Washington, U. S. Government Printing Office, 1959. Price, $3.75.

This definitive work on shock tubes lives up to the standards of excellence set by other sections of the *Handbook*. Intended as a practical reference for scientists and engineers concerned with the design and performance of supersonic vehicles, Section 18 is composed

of seven chapters, including a very brief introductory one. The six main chapters deal with: Performance of Simple Constant-Area Shock Tubes (one-dimensional rarefaction, plane shock waves, the wave system, etc.); Observed Flows in a Constant-Area Shock Tube (boundary layer effects, wave speed measurements); Production of Strong Shock Waves (attenuation, the diaphragm shock tube); Applications of the Shock Tube (as a wind tunnel, in aerophysics research, high-temperature gas physics); and Shock-Tube Flow Measurement and Instrumentation. An abundance of graphs, tabular material and diagrams supplements the text. A list of 114 references (all but eight covering the last twenty years) is included.

THE THEORY OF THIN ELASTIC SHELLS, edited by W. T. Koiter. 496 pages, illustrations, 6 × 9 in. New York, Interscience Publishers, Inc., 1960. Price, $9.00.

This volume is the proceedings of a symposium held in Delft, Holland, in August 1959. Sponsored by the International Union of Theoretical and Applied Mechanics, the symposium consists of 23 papers (17 in English, 5 in German and 1 in French) dealing with: the general equations of shell theory; buckling and nonlinear theory of spherical, cylindrical and conical shells; analysis of helicoidal shells; nonsymmetrical deformations of hyperboloic paraboloid, spherical and cylindrical shells; membrane theory of developable shells and of shells of positive curvature; and dynamics of thin shells.

INTRODUCTION TO LINEAR PROGRAMMING, by Walter W. Garvin. 281 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $8.75.

As indicated in the title, this is an introductory treatment of linear programming, intended as a text for a one-semester course. Eighteen chapters are divided into three main parts: The General Linear-Programming Problem; The Transportation Problem and Its Variants; and Special Methods. In Part I, the simplex method is developed, and details of the computational procedure are described. Part II covers a specialization of the simplex method, and devotes some space to such subjects as the caterer problem, the assignment problem, the tanker-routing problem, and the generalized transportation array. In Part III, the author deals with special methods and some of their applications, for example, the revised simplex method, parametric linear programming, duality and the warehouse problem. Several of the chapters (for example, the generalized transportation problem and statistical linear programming) have not, until now, been included in such a text.

NOISE REDUCTION, edited by Leo L. Beranek. 752 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $14.50.

Noise control is fast becoming a major problem for the nation, in aircraft, home appliances, automotive vehicles, airports, etc. In an effort to conquer the problem, M.I.T. offered summer courses in noise reduction in 1953, 1955, 1957 and 1960. The present volume draws heavily on the material presented at those summer courses. The book is divided into four parts covering sound waves and their measurement, fundamentals underlying noise control, criteria for noise and vibration control, and practical noise control. The book covers fundamental principles of sound transmission, techniques of acoustical measurement in several kinds of environment, methods of controlling noise, and case histories of practical noise control. The book can be read with profit by graduate engineers in nearly any technical field.

DIGITAL COMPUTERS AND NUCLEAR CALCULATIONS, by Ward C. Sangren. 208 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $8.50.

The author has written this book for nuclear scientists and engineers, in order to acquaint them with the uses of high-speed calculations in the design and application of nuclear reactors. The first four chapters form an introduction to nuclear reactors, digital computers, programming, and numerical analysis. The last four chapters discuss actual reactor calculations, under the following titles: A Code for Fission-Product Poisoning; Diffusion and Age-Diffusion Calculations; Transport Equation—Monte Carlo; and Additional Reactor Calculations (kinetics, burnout, etc.). A useful bibliography, arranged by chapters, also includes a list of nuclear code reports.

DESIGN OF STEEL STRUCTURES, by Boris Bresler and T. Y. Lin. 710 pages, illustrations, 5½ × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $9.75.

The application of theoretical mechanics to the solution of structural problems has proven of great value to engineers. In addition, improved structural materials and construction techniques have challenged the designer. This book presents a rational approach to the design of steel structures, correlating it with current design practice. The authors have written this book as a text for civil engineering courses; with judicious selection of material, the teacher can use it either in elementary or advanced courses. The student is assumed to have a knowledge of general principles of statics, mechanics of materials, dynamics, and some structural theory. The diagrams are unusually clear and uncluttered; the long lists of problems cover a wide range of difficulty. The twelve chapters cover general principles of structural design, elasticity and plasticity, safety, riveted connections, welds, bolted and pinned connections, tension and compression members, bending and torsion of beams, beams and girders, design of steel buildings and bridges, and light-gage members.

RIEMANN SURFACES, by Lars V. Ahlfors and Leo Sario. 382 pages, 6 × 9 in. Princeton, Princeton University Press, 1960. Price, $10.00.

This book has two purposes—to introduce the reader to the theory of Riemann surfaces and to set forth recent problems and results. Open surfaces (rather than closed) are emphasized. The first chapter is a brief, but thorough treatment of topological surfaces (including simplicial homology, singular homology, compactification, etc.). Chapter II, after defining Riemann surfaces and reviewing elementary theory of functions on Riemann surfaces, defines the Dirichlet problem and gives its applications. Chapter III on harmonic functions deals with normal and principal operators, principal functions, planar surfaces and capacity functions. In Chapter IV, classification theory is covered, with sections devoted to parabolic and hyperbolic subregions, modular and explicit tests, and applications. The final chapter, entitled "Differentials on Riemann Surfaces," deals with elementary properties of differentials, orthogonal projection, periods and singularities, and classical theory. The book is self-contained and the reader is assumed to have only a rudimentary knowledge of algebra and Hilbert space theory. Some topics, such as the generalization of the theorems of Runge, Weierstrass and Mittag-Leffler, are omitted, because the authors felt that they have been adequately treated elsewhere.

PHOTO-ELECTRONIC IMAGE DEVICES, edited by J. D. McGee and W. L. Wilcock. 397 pages, illustrations, 6 × 9 in. New York, Academic Press, Inc., 1960. Price, $12.00.

Volume XII of the Advances in Electronics and Electron Physics series, like the ninth volume on geophysical topics, departs from established policy by limiting the scope of the papers to one specialized topic—image tubes and related devices. The thirty papers (two in French) presented in the book are the record of a symposium held in September 1958 in London; the authors represent five nations—U. S. A., England, France, The Netherlands and Germany. The contributions cover the whole range of photo-electronic images (with the exception of solid state devices), and include image tube research programs, image converters, image intensifiers, field emission, the channeled image intensifiers, uses in nuclear physics and television-camera tubes, signal-to-noise ratio, an isophote converter, and X-ray devices. Discussions are included with some of the papers.

MATHEMATICAL METHODS AND THEORY IN GAMES, PROGRAMMING AND ECONOMICS, by Samuel Karlin. Two volumes. Volume I, 433 pages, 6 × 9 in. Vol. II, 386 pages, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1959. Price, $12.50 each volume.

Volume I covers matrix games, programming and mathematical economics, while Vol. II is on the theory of infinite games. Intended as texts for advanced undergraduates or graduates, the two books can also be used as references in the fields of management, economics, military tactics and operations research. The two volumes are independent of each other. Each volume contains a bibliography and each is adequately provided with graded problems, whose solutions are given in separate sections.

WORK SAMPLING FOR MODERN MANAGEMENT, by Bertrand L. Hansen. 263 pages, diagrams, 6 × 9 in. Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960. Price, $7.50 (trade); $5.65 (classroom).

This practical guide provides a new and economical tool for management in measuring work through easy-to-use, reliable sampling techniques. A step-by-step explanation is offered on how to make a profitable fact-finding sampling study from the initial planning stage to the last step of the follow-up. It also shows how modern management can solve many major problems presented by the current rising cost of indirect labor and increasing replacement of people by machines. The reader is then taken through the various stages of the sampling process at an easy pace to meet the needs of undergraduates, advanced students and persons in the field.

EXTRACTIVE AND PHYSICAL METALLURGY OF PLUTONIUM AND ITS ALLOYS, edited by W. D. Wilkinson. 314 pages, illustrations, 6 × 9 in. New York and London, Interscience Publishers, 1960. Price, $10.50.

This volume represents the papers presented at a symposium held in San Francisco, California in February 1959, and sponsored by the Nuclear Metallurgy Committee, Institute of Metals Division, and Titanium, Uranium and Uncommon Metals Committee, Extractive Metallurgy Division of The Metallurgical Society, American Institute of Mining, Metallurgical and Petroleum Engineers. There are fourteen papers included in the book, plus a special introduction and annotated bibliography by the editor.

THEORY OF DETONATION, by Ia. B. Zeldovich and A. S. Kompaneets. 284 pages, diagrams, 6 × 9 in. New York and London, Academic Press, 1960. Price, $10.00.

This book is the first systematic presentation of detonation theory based on papers completed at the Chemical Physics Institute. The papers give an explanation of the important effect of spin detonation, which is observed near the detonation threshold. Further, they have explained the nature and conditions of the change of combustion into detonation, and disclosed the extraordinary conditions under which a detonation is propagated in pipes with roughened walls, and they have explained the narrow limits of propagation of a detonation in smooth pipes.

## PUBLICATIONS RECEIVED

ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS, by George J. Thaler and Robert G. Brown. Second edition (formerly "Servomechanism Analysis"), 648 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $14.50.

THERMAL ENGINEERING, by Harry L. Solberg, Orville C. Cromer and Albert R. Spalding. Revised edition of "Elementary Heat Power," 649 pages, illustrations, 6 × 9 in. New York, John Wiley & Sons, Inc., 1960. Price, $9.50.

MECHANICS, by Keith R. Symon. Second edition, 557 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1960. Price, $10.50.

DICTIONARY OF AUTOMATIC CONTROL, by Robert J. Bibbero. 282 pages, 5 × 7¼ in. New York, Reinhold Publishing Corp., 1960. Price, $6.00.

LITTLE STAR, by W. S. Rozier. 60 pages, 8½ × 11 in. Milledgeville (Ga.), Old Capital Publishing Co., 1960. Price, $2.00 (paper).

INSTALLING HI FI SYSTEMS, by Jeff Markell and Jay Stanton. 224 pages, illustrations, 5½ × 8½ in. New York, Gernsback Library Inc., 1960. Price: $5.00 (hard cover); $3.20 (paper).

## NATIONAL BUREAU OF STANDARDS NEWS

### FADING RATE RECORDER

To aid in radio wave propagation studies, the National Bureau of Standards has devised an instrument, under the sponsorship of the U. S. Air Force, to record the rate at which received signals fade. Developed by B. B. Balsley and improved by W. B. Harding, R. J. Jansen, and J. W. Koch, all of the Bureau's Central Radio Propagation Laboratory at Boulder, Colo., the instrument records the number of times per second that a signal envelope crosses the average signal level with a positive slope. It records fading rates down to almost zero cycles per second, and as high as 300 per second.

Such irregularities in the atmosphere and ionosphere as humidity, temperature, pressure, and index of refraction can combine to cause more than one propagation path to exist between transmitter and receiver. Each path, traveling over a different route, will have a different length. If all the signals arrive in phase, the received signal will be strengthened; if they arrive out of phase, there may be cancellation and the signals will fade. This phenomenon is called multi-path fading. Comparison of fading rate records with signal intensity records obtained simultaneously provides a useful aid in interpreting simple mode propagation.

Although the instrument records the rate of average rather than median signal crossing, and hence may not strictly agree with fading rates as frequently defined, it does provide a convenient means for comparing average fading rates for various propagation conditions and transmission frequencies. The results of the fading-rate records must be applied cautiously to communications problems, as the depth of fading and amplitude distribution of the envelope vary considerably. The device gives no indication of these factors.

The recorder operates from the intermediate frequency output of a receiver that has a 12-sec or longer AGC time constant. The signal from the IF is applied to an integrating circuit with a 6-sec time constant to determine the average signal level. The continuously varying signal is compared to the average signal, and when it rises above the average level it turns on a Schmitt trigger circuit. When the signal drops below the average level, the trigger turns off. The square-wave output of the Schmitt trigger is differentiated, and the resulting positive pulse indicates when the signal crosses the average level in a positive direction.

This pulse output is integrated, amplified, and then applied to a strip-chart recorder. The voltage to the recorder is proportional to the number of one-shot pulses each second from the Schmitt trigger circuit; hence the recorder indication is a linear function of fading rate.

The fade rate recorder has proved to be quite stable in operation after the initial warm-up period, when used with regulated power supplies. Even though dc coupling is used between all stages up to and including the Schmitt trigger, the circuitry is designed so that negligible drift occurs in the critical operating voltages.

Flutter fading at rates up to 20 cps has been noted in typical recordings for radio paths through the auroral zone. Other recordings show a change in fading rate when the signal level suddenly changes; probably the propaga-

tion mode has changed at these times. Spikes on fade records have occurred even when the transmitter is off; these spikes are probably caused by noise fluctuations. Occasional noise spikes are noted on NBS radio station WWV recordings. These spikes appear to result from impulse noise interference.

### HIGH-RESOLUTION SCANNING ANTENNA

An antenna developed by the National Bureau of Standards promises to provide science with a new tool for experimental studies of the ionosphere. Without benefit of electric or mechanical moving parts, it scans a 42-deg. azimuthal arc and can determine immediately the direction of radio signals received from a "forward scatter" transmission.

In this type of very high frequency (VHF) radio propagation, radio energy is scattered from cloud-like concentrations of electrons in the lower ionosphere and may arrive at the receiver site from any number of directions. The new development can pinpoint instantly the direction from which the scattered signal arrives and could prove invaluable in tracing "best paths" for forward-scatter communication. It can be adapted for use in the observation of a number of other ionospheric phenomena.

At Table Mesa, a field station of the NBS Boulder Laboratories, H. V. Cottony and Alvin C. Wilson, of the Antenna Research Section, Radio Systems Division, have erected a row of seven 5-element Yagi antennas, directed along the Great Circle Path to the NBS Long Branch field station transmitter in Havana, Ill.—800 miles distant from Boulder. The number of elements in the array is now being increased to 25 to permit a beam width of only 1.5 deg.

Radio signals beamed from the Havana station at a frequency of 41 Mc into the ionosphere are received at Table Mesa, but, heretofore, the direction from which the signal was reflected by the electron "clouds" has remained unknown.

The electronic rapid scanning device, developed for use with the moderately directive Yagi antennas, swings a 5.8-deg. beam in azimuth through the 42-deg. sector 20 times a second. An oscilloscope synchronized to the incoming signal permits direct viewing or photographing. The device is novel in that it operates by purely electronic control of phasing and involves no mechanical apparatus such as that used for the sweep of radar antennas.

The ionospheric forward-scatter mode of radio propagation has distinct advantages in that it is resistant to the polar blackouts which so often cause complete fading in other modes of short-wave propagation. Up to the present there has been no method for determining the path of the received signal, but now the direction of arrival of an ionospheric scatter signal can be seen—and followed as the signal path shifts.

Besides providing a method for study of long-distance ionospheric forward-scatter communication phenomena, the antenna will be a valuable tool for other experimental studies. Bearings of ionized meteor trails may be obtained instantly. The principle is applicable to a design of an antenna array for riometers (devices that listen to extra-terrestrial, or cosmic, noise which partially penetrates the ionosphere). It could be used as a small-space, multi-direction antenna for receiving and—with design changes—for transmitting. It is considered to have a number of possible uses in many of the research programs now probing toward outer space.

# THE FRANKLIN INSTITUTE

## MINUTES OF THE STATED MEETING

**October 19, 1960**

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in Franklin Hall, following the Annual Medal Day Reception and Dinner, at which approximately 420 members and guests were present.

Mr. Wynn Laurence LePage, President, called the meeting to order and announced that the minutes of the April Stated Meeting had been printed in the May issue of the JOURNAL and the minutes of the May Stated Meeting had been printed in the June issue of the JOURNAL. There being no corrections or additions, the minutes were approved as published.

The President stated that for many years it has been the custom of the Board of Managers of The Franklin Institute to propose The Franklin Medalist for election to Honorary Membership in The Franklin Institute, and at the Stated Meeting on Wednesday, May 18, 1960, the membership of The Institute elected Dr. Roger Adams to Honorary Membership in The Franklin Institute, effective October 1960. Mr. LePage then presented to Dr. Adams the Certificate of Honorary Membership in The Franklin Institute of the State of Pennsylvania.

The President then stated that on September 15, 1960, certain proposed changes in the By Laws of The Franklin Institute were mailed to all Voting Members of The Institute, which is in accordance with the provision of the By Laws. He called for a motion on these proposed amendments, and a motion was made by a voting member of The Institute, who presented the following resolution:

> RESOLVED, that the By Laws of The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts be amended to read as set forth as part of the Notice to Voting Members, dated September 15, 1960, and that these amendments become effective as of this date, October 19, 1960.

This motion was duly seconded by a voting member of The Institute. The President then called for discussion.

Following a discussion on this resolution, a motion was made by a voting member of The Institute to postpone action on the proposed By Laws' changes until the next Stated Meeting of The Institute. After a short discussion on this second motion, the President asked the member who had moved for the adoption of the resolution to amend the By Laws and the member who had seconded this motion if they had any objections to a vote being taken on the second motion. As there was no objection, the President asked for a second on the secondary motion. This motion was seconded by a voting member. The President asked the members to rise who were in agreement to postponing action on the proposed amendments until the next Stated Meeting of The Institute. He then asked the members who were not in agreement with this postponing action to rise. The rising vote indicated the majority of the voting members were in favor of postponing action on the proposed amendments to the By Laws until the next Stated Meeting of The Institute. The President then declared that action on the proposed amendments to the By Laws, which had been mailed to all voting members of The Institute on September 15, 1960, would be postponed until the next Stated Meeting of The Institute to be held on Wednesday evening, November 16, 1960.

The business meeting being completed, preparatory to the presentation of Medals, the President reviewed the plans for the coming year in The Institute, emphasizing that true to its tradition The Franklin Institute is moving at an accelerated pace toward specific and urgent goals set for us by national circumstances and inspired by The Institute's heritage of leadership in science education.

The President then recognized the Sponsors of the Medalists, who presented the recipients of the Awards. The presentation of the Medals was made by President LePage.

Following the presentation of Medals, the President presented the speaker of the evening, Dr. Roger Adams, The Franklin Medalist and Research Professor of Chemistry, Emeritus, University of Illinois. Dr. Adams' talk on "Organic Chemistry in the Space Age" was enthusiastically received.

The President expressed warm and sincere thanks to Dr. Adams for his excellent talk and thanked all present for a full and enjoyable evening.

The meeting was adjourned at 10:30 P.M.

WILLIAM F. JACKSON, JR.
*Secretary*

NOTE: The entire proceedings of Medal Day will be published in the December issue of the JOURNAL OF THE FRANKLIN INSTITUTE.

# 1960 MEDALS

## Cresson Medals (1848)

### Hugh L. Dryden

Deputy Administrator
National Aeronautics and Space Administration
Washington, D. C.

### Arpad Ludwig Nadai

Formerly Consulting Mechanical Engineer
Westinghouse Research Laboratories
Pittsburgh, Pa.

### William Francis Gray Swann

Director Emeritus
Bartol Research Foundation
Swarthmore, Pa.

## Certificate of Merit (1882)

### Richard R. Moore

Chief Metallurgist
Naval Air Material Center
Philadelphia, Pa.

## Longstreth Medals (1890)

### W. Edward Chamberlain

Special Assistant to the Chief Medical Director
for Atomic Medicine
Veterans Administration
Washington, D. C.

### John W. Coltman

Associate Director, Research Laboratory
Westinghouse Electric Corporation
Pittsburgh, Pa.

### Frederick A. Keidel

Research Project Engineer
Experimental Station
E. I. du Pont de Nemours & Company
Wilmington, Del.

### Moulton B. Taylor

President
Aerocar, Inc.
Longview, Wash.

## Potts Medal (1906)

### Charles Stark Draper

Director of Instrumentation Laboratory
Massachusetts Institute of Technology
Cambridge, Mass.

## Levy Medals (1923)

### Ezra S. Krendel

Manager, Engineering Psychology Laboratory
The Franklin Institute Laboratories
Philadelphia, Pa.

### Duane T. McRuer

President
Systems Technology, Inc.
Inglewood, Calif.

## Henderson Medals (1924)

### Ernest Chatterton

Engineering Consultant
London, England

### Herbert Sammons

Managing Director
D. Napier and Son, Limited
London, England

## Wetherill Medals (1925)

### Raymond Castaing

Staff Member
Laboratoire de Physique des Solides
Université de Paris
Paris, France

### Walter Juda

Executive Vice President
Ionics, Incorporated
Cambridge, Mass.

### Victor Vacquier

Research Geophysicist
Scripps Institute of Oceanography
La Jolla, Calif.

## Clark Medals (1926)

### Robert W. Cook

Vice President
C. W. Fuelling, Inc.
Decatur, Ind.

### Lee F. McBride

C. W. Fuelling, Inc.
Decatur, Ind.

## Brown Medal (1938)

### R. Buckminster Fuller

Structural Designer
Forest Hills, N. Y.

## Ballantine Medals (1946)

### Rudolf Kompfner

Director of Electronics and Radio Research
Bell Telephone Laboratories, Inc.
Holmdel, N. J.

### John R. Pierce

Director of Research—Communications Principles
Bell Telephone Laboratories, Inc.
Murray Hill, N. J.

### Harry Nyquist

Formerly Assistant Director of Systems Studies
Bell Telephone Laboratories, Inc.
Murray Hill, N. J.

## Franklin Medal (1914)

### Roger Adams

Research Professor of Chemistry, Emeritus
University of Illinois
Urbana, Ill.

## NEW BOOKS IN THE FRANKLIN INSTITUTE LIBRARY

### MATHEMATICS

Conference on Analytic Functions, edited by R. Nevanlinna. Princeton University Press, 1960.

Boundary Problems in Differential Equations, by R. E. Langer. University of Wisconsin Press, 1960.

Naive Set Theory, by P. R. Halmos. Van Nostrand, 1960.

Special Functions, by E. D. Rainville. Macmillan, 1960.

Mathematical Methods and Theory in Games, Programming, and Economics, by S. Karlin. Addison-Wesley, 1959.

Mathematics For Communication Engineers, by S. J. Cotton. Macmillan, 1959.

Engineering Mathematics, by J. Blakey. Philosophical Library, 1960.

Probability and Statistics, by U. Grenander. Wiley, 1959.

University Mathematics, by J. Blakey. 2d ed. Philosophical Library, 1959.

High Speed Computing Methods and Applications, by S. Hollingdale. English Universities Press, 1959.

Classical Mathematics, by J. Hofmann. Philosophical Library, 1960.

Elements of the Theory of Markov Processes and Their Applications, by A. T. Bharucha-Reid. McGraw-Hill, 1960.

Probability and Related Topics in Physical Sciences, by M. Kac. Interscience, 1959.

### PHYSICS

Nuclear Physics and Atomic Energy, by G. J. Béné. Elsevier, 1960.

Fast Reactions in Solids, by F. P. Bowden. Interscience, 1960.

Quantum Theory of Atomic Structure, by J. C. Slater. McGraw-Hill, 1960.

Optics in Metrology, by P. Mollet. Pergamon, 1960.

International Congress on Vacuum Techniques, by E. Thomas. Pergamon, 1960.

Electrical Noise, by D. A. Bell. Van Nostrand, 1960.

Unsteady Motion of Continuous Media, by K. P. Staniukovich. Pergamon, 1960.

Non-Linear Wave Mechanics, by L. de Broglie. Elsevier, 1960.

Correspondence of Sir Isaac Newton, edited by H. W. Turnbull. Cambridge University Press, 1959.

The Electric Arc, by J. M. Somerville. Wiley, 1959.

Masers; Microwave Amplification and Oscillation by Stimulated Emission, by G. Troup. Wiley, 1959.

Electron Physics, by O. Klemperer. Academic Press, 1959.

Fluctuation Phenomena in Semi-Conductors, by A. Van der Ziel. Academic Press, 1959.

Evaporation and Droplet Growth in Gaseous Media, by N. A. Fuchs. Pergamon, 1959.

Plasma Physics, by J. G. Linhart. Interscience, 1960.

Neutronen, by N. A. Vlasov. Hoffman Verlag, 1959.

Introduction to Colloid Chemistry, by K. J. Mysels. Interscience, 1959.

Collected Scientific Papers of Paul Ehrenfest, edited by M. J. Klein. Interscience, 1959.

Trends in Atomic Physics, by O. R. Frisch. Interscience, 1959.

The Special Theory of Relativity, by J. Aharoni. Clarendon Press, 1959.

International Symposium on Stress Wave Propagation in Materials, edited by N. Davids. Interscience, 1960.

Deformation Strain & Flow, by M. Reiner. 2d ed. Interscience, 1960.

An Introduction to Radiation Counters and Detectors, by C. Washtell. Philosophical Library, 1960.

Engineering Thermodynamics, by D. B. Spalding. McGraw-Hill, 1959.

### CHEMISTRY AND CHEMICAL ENGINEERING

Frozen Free Radicals, by G. Minkoff. Interscience, 1960.

Gas Purification, by A. L. Kohl. McGraw-Hill, 1960.

STYLE GUIDE FOR CHEMISTS, by L. F. Fieser. Reinhold, 1960.
CHEMISTRY OF NUCLEAR POWER, by J. K. Dawson. Newnes, 1959.
TRANSPORT AND ACCUMULATION IN BIOLOGICAL SYSTEMS, by E. J. Harris. 2d ed. Academic Press, 1960.
VALENCY, CLASSICAL AND MODERN, by W. G. Palmer. 2d ed. Cambridge University Press, 1959.
DETOXICATION MECHANISMS, by R. T. Williams. Wiley, 1959.
THE PROTON IN CHEMISTRY, by R. P. Bell. Cornell University Press, 1959.
PRINCIPLES OF OPTICAL CRYSTALLOGRAPHY, by A. V. Shubnikov. Consultants Bureau, 1960.
THEORY OF UNIMOLECULAR REACTIONS, by N. B. Slater. Cornell University Press, 1959.
INTERNATIONAL CONFERENCE ON STRUCTURE AND PROPERTIES OF THIN FILMS, edited by C. A. Neugebauer. Wiley, 1959.
THE SEQUESTRATION OF METALS, by R. L. Smith. Chapman & Hall, 1959.
INSTABILITY CONSTANTS OF COMPLEX COMPOUNDS, by K. B. Yatsimirskii. Consultants Bureau, 1960.
X-RAY ABSORPTION AND EMISSION IN ANALYTICAL CHEMISTRY, by H. A. Liebhafsky. Wiley, 1960.
MODERN COORDINATION CHEMISTRY, by J. Lewis. Interscience, 1960.
HETEROMETRY, by M. Bobtelsky. Elsevier, 1960.

### ELECTRICAL ENGINEERING

THE DYNAMIC BEHAVIOR OF THERMOELECTRIC DEVICES, by P. E. Gray. Technology Press, 1960.
APPLICATIONS OF THERMOELECTRICITY, by H. Goldsmith. Wiley, 1960.
INTERNATIONAL SYMPOSIUM ON THE THEORY OF SWITCHING. Harvard University Press, 1959.
SOLID STATE PHYSICS IN ELECTRONICS AND TELECOMMUNICATIONS, edited by M. Désirant. Academic, 1960.
TRANSFORMERS AND GENERATORS FOR POWER SYSTEMS, by R. Langlois-Berthelot. Philosophical Library, 1960.
MODERN ELECTRONIC COMPONENTS, by G. W. A. Dummer. Philosophical Library, 1959.
THE CATHODE-RAY TUBE AND ITS APPLICATIONS, by G. Parr. 3rd ed. Reinhold, 1959.
SYMPOSIUM ON MICROMINIATURIZATION OF ELECTRONIC ASSEMBLIES, edited by Eleanor F. Horsey. Hayden, 1959.
CONTROL SYSTEMS ENGINEERING, edited by W. W. Seifert. McGraw-Hill, 1960.
INTRODUCTION TO MODERN NETWORK SYNTHESIS, by M. E. Van Valkenburg. Wiley, 1960.
FEEDBACK CONTROL SYSTEM ANALYSIS AND SYNTHESIS, by J. J. D'Azzo. McGraw-Hill, 1960.

### SPACE SCIENCE

ASTRONOMICAL PHOTOGRAPHY AT THE TELESCOPE, by T. Rackham. Faber and Faber, 1959
INTERNATIONAL CONGRESS FOR THE AERONAUTICAL SCIENCES, edited by Th. von Kármán. Pergamon, 1959.
TURBULENT FLOWS AND HEAT TRANSFER, by C. Lin. Princeton University Press, 1959.
FIGURES OF EQUILIBRIUM OF CELESTIAL BODIES, by Z. Kopal. University of Wisconsin Press, 1960.
AERONAUTICAL DICTIONARY, by F. Davis Adams. U. S. Gov't Print. Off., 1959.
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THEORY OF INERTIAL GUIDANCE, by C. L. McClure. Prentice-Hall, 1960.

### MATERIALS SCIENCE

A HISTORY OF METALS, by L. Aitchison. Interscience, 1960.
THE MARINE CORROSION HANDBOOK, by T. Rogers. McGraw-Hill, 1960.
HIGH SPEED TESTING, by A. G. H. Dietz. Interscience, 1960.
CORROSION OF METALS, by U. R. Evans. 2d ed. St. Martins Press, 1960.

## CURRENT TOPICS

**Adaptive Waveform Recognition.**—The development of a new system for detecting and classifying unknown or concealed signals has been announced by the General Electric Research Laboratory. "The new system has detected signals buried in random noise that could not be detected by trained observers, even when they knew what to look for," according to Dr. Guy Suits, General Electric vice president and director of research. "Moreover, the system can detect these buried signals even when their full characteristics are previously unknown, and in the process of detection it determines those characteristics," Suits added.

"With but limited prior knowledge of the characteristics of the signal it is supposed to detect," according to Suits, "the system can, with experience, separate the signal from the random noise in the input data. It does this by associating those elements or the input data which are similar, and modifying itself to detect optimally these similarities," Suits said.

Details of the new system which is called "adaptive waveform recognition," were presented at the Fourth London Symposium on Information Theory, at the Royal Institution, London, England. The system was developed by R. L. Shuey, C. V. Jakowatz, and G. M. White of the General Electric Research Laboratory.

The essential element of the adaptive waveform (or signal) recognition system is a self-adapting matched filter. This element learns with experience to adjust its impulse response so that it automatically forms a matched filter for the signal. The system can be extended to handle a multiplicity of fixed signals.

The capabilities of the self-adapting filter can be explained by considering the following situation. The filter is confronted by a waveform or signal that is fixed in shape (or perhaps changing slowly), that occurs randomly in time, and is buried in random noise. The filter has essentially no knowledge of the signal shape or its time of occurrence. It knows only that the signal is in a given bandwidth. In this situation, the filter is capable of detecting the presence of the signal, obtaining, with experience, a better estimate of what the signal is, and adapting itself to changes in the signal.

Applications of the new system are expected to be primarily in the various detection and classification areas involved in defence. A number of applications in the general area of information handling may be possible through special adaptations of the system.

**Complete Electronic Banking Equipment Installed.**—A vast, completely new, fully automatic electronic bank bookkeeping system manufactured by Burroughs Corporation—the first of its kind in the world, according to the manufacturer—went into operation this fall at The First Pennsylvania Banking and Trust Company in Philadelphia, oldest bank in the U. S.

Designed, developed and built by Burroughs Corporation, the system consists of intricate sub-systems linked together into a master system that ultimately will perform all of the bank's major accounting functions automatically through electronics.

The system is presently being utilized for automatic check handling, but it is capable also of processing regular checking accounts. Revolving credit, installment loans and mutual funds are currently being processed on the computer system. Personal and corporate trusts, payroll, management reports, the general ledger including expense distribution and other accounting operations are being studied for future application.

Check-out of the system is being conducted on the bank's special checking accounts which brings into operation the latest of the sub-systems to be installed—the B 301 Magnetic Document Processing System, an entirely new Burroughs development.

The B 301 system uses MICR (Magnetic Ink Character Recognition), the common machine language principle as selected by the American Bankers Association for use throughout the United States. Thus it uses actual MICR encoded media (checks and deposit tickets) as the input "trigger" of the system.

With the addition of a B 301 to the Burroughs 220 computer, the master system can operate directly from MICR encoded media as well as from punched paper tape, punched cards, or magnetic tape input.

The list of accomplishments of the B 301 system alone is impressive. Operating from pre-coded MICR customer checks, it will electronically read the checks, edit the coded data, sort, accumulate totals, classify, print listings and convert acceptable information to magnetic tape for subsequent handling by other sub-systems, including the 220 computer and an ultra-high speed printer.

At speeds of 750 lines of 120 characters per minute (maximum capability of 1,500 numerical lines per minute) statements are prepared by the high speed printer at 1,400 statements per hour. The check processor-reader used in First Pennsylvania's system—the fastest in the world—reads and sorts items automatically at speeds up to 1,560 items per minute.

The B 301 system is the fastest, most versatile method available for converting check information or source documents to magnetic tape. While operating, it provides complete automatic item control, converting for input to the computer only totally acceptable in-balance data.

These are the sub-systems linked together to form the master system:

(a) Document encoding and proof system.
(b) Document sorter and processing system.
(c) Sorter-lister-converter system.
(d) Data processing computer with magnetic tape units.
(e) High speed printer system.

A step-by-step review of how the system operates to automate the handling and processing of special checking accounts can give a clue to the efficiency First Pennsylvania can expect to gain when the system is applied to other operations.

Customer checks—encoded with an account number and bank routing symbol before being issued to the customer—are received at the bank and run through a Burroughs P 703 Amount and Account Number printer. The P 703 provides both proof and amount encoding in a single operation for such "on us" items as clearings and Federal Reserve letters. Other "on us" items already proved may also be encoded with MICR by the machine.

Once fully coded and proven, the checks are fed into the B 101 Sorter-Reader, which in reality is the B 301 system's input, sorting the checks

while at the same time transmitting to a control unit the data from the checks.

While co-ordinating operations of the Sorter-Reader with other units of the B 301 system—Electrostatic Multi-Printer and Magnetic Tape Units—the control unit simultaneously edits and tests the incoming data to determine item acceptance and distribution. It also accumulates totals for settlement and control, directs sorting of the items and selects the pockets into which they fall. It further controls the form in which the data are fed into the electrostatic printer for listing information.

The control unit part of the B 301 system directs acceptable item information to magnetic tape for processing through the 220 computer. The tape is used for updating customer's accounts and, subsequently, the preparation of customer statements—all without human intervention.

Output is provided by the Electrostatic Multi-Printer in the form of a master listing tape and three other tapes for listing "on us" items, unidentified items and exceptions.

The master list printed by the printer provides a record of every item processed by the Sorter-Reader in the same batch sequence in which it arrives from a clearing house or Federal Reserve bank. In addition, the list shows the batch number, source code, cash letter total, customer account number and the amount of every item processed. When all items have been processed, the batch total is automatically balanced with the total of the items and either a zero settlement or the difference is printed on the master tape before the next batch is automatically processed.

The electrostatic printing technique records data as fast as the Sorter-Reader operates—1,560 items per minute. However, at the end of each batch run, settlement data are printed out on the master tape at the rate of 3,000 words per minute. Only fractions of a second elapse before settlement of the first batch and the beginning of the next batch operation.

The magnetic tape provided as an automatic by-product of B 301 system operation is the vital "link" between this system, the 220 computer and the high speed printer which produces the finished customer statements.

First Pennsylvania's new master automation system has the "built in" ability to perform all of the bank's major data processing with the same speed, accuracy and safeguards it brings to check handling. The system achieves a notable first in being the leader in linking live media processing with high speed computing from multiple methods of input.

**Plastic Cocoons for Return to Earth.**—How a man in space could "spin his own cocoon" for a fast return to Earth in an emergency was described by Harold L. Bloom, a systems engineer at General Electric's Missile and Space Vehicle Department in Philadelphia, in a technical paper presented at the 11th International Astronautical Congress in Stockholm. The paper was co-authored by John H. Quillinan, also a Department systems engineer, who created the "cocoon" system.

Made of plastic foam, the "cocoon" would be formed about the man as he started on his return trip. It would serve to protect him from the heat caused by the friction of the atmosphere and also cushion him against the landing shock.

Here is how the system would work:

When it became necessary to abandon his space craft, the man would don a space suit (if he didn't already have it on) and with it an oxygen supply,

recovery aids, and survival gear. Over the space suit would be a plastic covering with attached tanks of foaming plastic and mixer. The man would also be equipped with a retro-rocket.

First the man would fire the rocket to head him back to a reasonable location on Earth. He then would inflate the plastic covering which would have the proper shape for re-entry and impact. Next the space between the man and the plastic covering would be filled with foam plastic. A hardened dense plastic foam would serve as the heat shield and a less dense spongelike foam would form a cushion for the man.

During re-entry into the atmosphere the dense foam would ablate (be changed by heat into gaseous form) and protect the man from the heat. In the atmosphere a parachute would be released to slow down the man and the vehicle during the fall to Earth. At about 30,000 feet, the parachute would pull cutting cords to remove the light weight foam plastic from around the man's hands and arms. After landing the man would release himself from the light weight foam by pulling on cutting cords with his freed hands and arms. He then could obtain the survival equipment embedded in the plastic foam. Should he land on water, the vehicle would serve as a raft.

Safely returned to Earth, survival of the man will depend on being quickly located.

"One of the most important search and recovery considerations will be the monitoring of an emergency beacon frequency by a world-wide facility of radio stations. Such a signal will probably be the first indication of a man re-entering from space under emergency conditions."

In addition to the signal from the emergency beacon, high intensity flares may be fired before re-entry and during re-entry radar chaff may be ejected to provide a large radar target. If the landing is on water, sofar bombs may be ejected to provide location information. Meanwhile, the radio beacon should continue to operate for several days after the landing to provide direction-finding information.

**Tracking Signals from "Echo" Satellite.**—Two space-borne radio beacons, each about the size and shape of an ordinary dinner plate and equipped with a half-ounce radio transmitter no larger than a cigarette lighter, are sending back the signals that enable ground stations to track the nation's new "Echo" balloon satellite as it circles the Earth. The radio beacons were specially developed by the Radio Corporation of America for the National Aeronautics and Space Administration.

The two beacons are the only operating electronic devices carried on the nation's newest satellite, which was launched from Cape Canaveral by NASA. Installed on opposite sides of the 100-ft. balloon, the transmitters broadcast a steady tone as "Echo" moves in orbit. Their signals provide a constant radio "fix" for the directional ground antennas that are being used experimentally to communicate from point to point by reflecting messages from the balloon's aluminized surface.

The heart of each beacon, and the source of the signals, is the half-ounce transistorized transmitter, one of the most compact yet developed for use in space. The transmitter and a miniature storage battery are embedded in molded foam plastic to form the complete assembly—an 11-ounce disk 10 in. across and only $\frac{3}{4}$ in. thick, designed to fit snugly against the huge balloon. On its upper surface, each

disk carries an array of 70 solar cells to produce power from sunlight, and a circular groove holding a slender antenna.

The device was developed by John C. McCubbin and Harold B. Goldberg, engineers at the Space Center of the RCA Astro-Electronics Division at Princeton. The engineers explained that the disk-shaped units were designed to be attached to the skin of the collapsed balloon before its insertion in the launching container. When the balloon was ejected from the container and automatically inflated to its full 100-ft. diameter, the antenna on each of the units sprang erect from the groove on the surface of the disk, and each of the transmitters was automatically switched on.

The engineers pointed out that the combination of solar cells and rechargeable storage batteries in the beacons provide power in both sunlight and shadow as "Echo" circles the Earth in its orbit. They explained that the solar cell array on either of the two units produces enough energy to power both transmitters and to recharge the batteries.

The two beacons are linked together by thin printed wiring that circles the inflated balloon, so that both transmitters receive power from a solar cell array when only one of them faces the sun. The transmitters radiate over 5 milliwatts of power at frequencies of 107.94 and 107.97 megacycles.

**Filter-Separator for Aviation Fuel.** —A lightweight, portable filter-separator for removal of water and solid contaminants from aviation and jet fuel is expected to materially reduce the hazard of engine failures occurring in light U. S. Army aircraft as a result of contaminated fuel.

Under development by the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va., the proposed unit, when completed, will be used in conjunction with the standard hand-operated A4C barrel pump for fueling aircraft directly from 55-gal. drums in the field. It has a capacity of 20 gal. per minute of contaminant-free fuel.

The filter-separator weighs 50 lb., is 30 in. long and 8 in. in diameter. The prototype was fabricated by the Warner Lewis Division of the Fram Corporation, Tulsa, Okla., under contract with the Laboratories.

Procurement of 25 models for field test is currently under way. These will be delivered to several Army aviation units for evaluation under actual operational conditions. If the field tests bear out the engineering test results, the 20 gpm filter-separator will be made military standard.

**Quick-Disconnect Terminal Strip.** —Infrared Industries, Inc., of Waltham, Mass., has developed a new quick-disconnect terminal strip which will greatly speed up the maintenance of electronic systems by making it possible to disconnect rapidly a malfunctioning unit from a system without time-consuming unwiring of inter-system connections.

Unlike any other terminal strip manufactured, the new IRI product is a combination of a conventional-design barrier terminal strip and a blade-type inter-connecting strip. The conducting contacts of the basic strip are slotted to accommodate the blades of the inter-connecting strip, which has only a single row of screw terminals. When connected, the two units form a barrier terminal strip with three rows of common terminals rather than the conventional two. By using the single row of terminals of the inter-connecting strip for all power and control signals, a malfunctioning unit can

be disconnected rapidly from the rest of the system by simply pulling out the plug-in strip. The unit can then be replaced with minimum system down-time.

The use of Everdur metal, a special alloy of bronze, used extensively in marine applications, makes the new IRI product the most rugged and completely corrosion resistant terminal strip available commercially. It is particularly useful for severe environmental applications. Where the quick replacement of units is required, the rugged positive electrical contact permits considerable contact misalignment. For example, the blades can be inserted in the basic terminal strip slots misaligned up to 30° without loss of positive contact.

**Cultivation of Algae in Heavy Water.**—An exhibit demonstrating the the cultivation of green algae in heavy water was shown by Argonne National Laboratory, Argonne, Ill., at the Seventh International Soil Science Congress at the University of Wisconsin.

This exhibit demonstrated, for the first time, how algae can grow in an environment where nearly all of the atoms of hydrogen have been replaced by atoms of the rare isotope deuterium, also known as heavy hydrogen. The original experiment, conducted by Dr. Joseph J. Katz of Argonne's Chemistry Division, was described in an article in the July issue of *Scientific American* magazine.

Deuterium is present to the extent of one part in five thousand parts in ordinary water. Deuterated water, or heavy water, is separated and concentrated on a large scale by the U. S. Atomic Energy Commission. Deuterium is not radioactive.

The exhibit consisted of a transparent plastic container 24 in. in diameter with a rotating paddle-wheel agitator. Nutrient, light, and a mixture of 5 per cent carbon dioxide and 95 per cent nitrogen gas was provided. The algae were kept cooled to room temperature.

Algae grown in heavy water synthesize organic compounds containing only deuterium in positions normally occupied by hydrogen. The deuterated compounds, for example, glucose, can then be fed other plants and animals in research to determine the effects of heavy hydrogen on metabolism. The preparation of many of these synthetic compounds by other laboratory methods is difficult, often impossible.

The design of the culture vessel exhibited at Madison prevents algae from settling to the bottom, permitting exposure of each cell of the culture to illumination. The apparatus is an efficient means of utilizing heavy water for the growth of algae, producing deuterated algae on the scale of ounces of algae per week.

The aim of the meeting was to "promote peace and health by alleviating hunger through soil science."

**Computers to Learn English.**—A programming system that employs simple English words to instruct an electronic computer—rather than the complicated numerical jargon now understood only by computer specialists—has been developed by the Radio Corporation of America. The system has been designed for use initially on the company's 501 computer.

The development was described by Donald H. Kunsman, Vice President and General Manager, RCA Electronic Data Processing Division, as "one of the most important steps yet taken in the computer industry." He said it would cut in half the time required to write programs—the instructions that tell a computer what to do

and when to do it—and would provide a tremendously valuable flexibility by permitting the English instructions to by used subsequently on not one but all of the RCA data-processing systems. This precludes time-consuming reprogramming when it becomes necessary to shift data processing from one type system to another, either temporarily or permanently. It will be especially helpful to companies with multiple installations.

The new system is called the RCA 501 COBOL Narrator. COBOL (Common Business Oriented Language) was created by a committee of computer manufacturers at the request of the Department of Defense, largest single computer user in the country, in order to provide a standardization language that would be interchangeable among systems of various manufacturers. RCA is the first company to implement the language.

At present a staff of experts—systems analysts and programmers—must spend weeks and months interpreting the needs of management for the computer and devising its work load. And it takes six months or more to train these people in computer techniques. The new system, Mr. Kunsman said, will shorten the training time to as little as one week.

The system operates in this fashion: The computer is programmed to accept, in place of the present complex numerical code, words chosen by the user that are related to the business involved, such as "payrole file," "wage," "employee number," "tax," etc. It is also programmed to accept some twenty verbs that act as commands —such as "write," "read," "divide," "add," "display" or "move"—and a number of conjunctives that link the verbs to the English nouns. The word "if," for example, tells the machine to make comparisons of data and to execute logical decisions as a result of those comparisons.

The words are punched on paper tape by means of a machine resembling a typewriter. They are then transferred to the magnetic tape and fed into the computer for action. By means of COBOL, the English commands are translated into machine instructions and the computer does what it is told to do.

Substituting English for numerals, the new system permits the user to write an instruction such as "SUBTRACT DEDUCTIONS FROM GROSS GIVING NET," whereas under the old system this would have to be written: "72-010237-00-600000" and two other numbers equally long.

Users of the RCA 501 already are training personnel and writing programs with the RCA 501 COBOL Narrator in preparing to take full advantage of the system as soon as it is available. Similar systems for the compact 301 and the big 601 are being developed and will be ready in 1961.

Present "machine coded" computer programs mean nothing to the person who has not made an intensive study of the machine for which the coding was written, Mr. Kunsman pointed out. However, COBOL programs are written in English, plain to everyone.

Mr. Kunsman said the development of COBOL is important for the following reasons:

"First, it provides faster and more economical techniques that are inherent in using the English language in programming rather than the complex numerical language.

"Second, it advances us toward the ultimate goal of complete compatibility with all computers.

"Third, automatic programming will, for the first time, give management practical control of the problems it wants solved through data proces-

sing—a factor especially valuable in those areas where system computations provide the basis for important management decisions."

RCA feels that the COBOL system will make it possible for customers to complete all the programs needed for efficient data processing between the time the data processing equipment is ordered and the delivery of the equipment, Mr. Kunsman said. Under current "machine coding" techniques there often is a considerable lag between the delivery of equipment and the completion of necessary programs.

The Department of Defense—which uses computers made by many manufacturers—sought in 1959 to end the confusion resulting from the difference in computer languages, which could not be understood by other systems. An organization called CODASYL, for Conference on Data Systems Languages, was established. It was this organization that developed the common language that is expected eventually to be implemented by virtually all computer manufacturers.

The RCA 501 COBOL Narrator system is now ready for use with RCA's 501 electronic data processing system and is applicable to the new RCA 301 and 601 systems.

**Caulking Compound Used in Radiation "Vacuum Cleaner."**—A unique "vacuum cleaner" duct system with octopus-like arms, especially designed to replace "used air" with fresh air in rooms of the Radiation Laboratories at Livermore, Calif., was made leakproof with the help of the kind of sealant and caulking compound normally used for such purposes as sealing swimming pools and keeping rain out of skyscrapers.

Facts about the debut of a sealant and caulking compound in the highly critical preserves of an atomic energy installation have been disclosed by Sun Chemical Corporation. Sun's A. C. Horn Companies West Coast Division helped to solve the leakage problem by prescribing the caulking compound, after conventional sheet metal joining techniques failed under the stress of alternating compressed air and vacuum.

The Livermore Laboratory's duct system, consisting of six large galvanized steel ducts 80 to 180 ft. long, was designed and installed to pull air from small rooms in the laboratory, push that air through radiation-filtering units, and constantly replace the withdrawn air with fresh air. The whole system came into being when officials of the Livermore facilities became concerned about the possible dangers of an escape of radioactive material. Such radioactive emissions, it was felt, might seep undetected into all parts of the laboratory unless a powerful vacuum system, constantly operating, was on hand to suck up any dangerous material before it could spread. It was emphasized that no accidents had actually happened; the system is precautionary.

In designing the duct system, the contractor, A. R. Peterson & Son of Hayward, Calif., first subjected test sections to a 24-hr. compression and expansion test with pressures at 4 psi. Joints between sections of the duct were made in the conventional form, with abutting ends of the sheet metal turned at right angles so that strips of the metal faced each other, with the longer of the two pieces bent over and around the adjoining piece for a tight seal. Lengths of angle iron were backed up against each side of the joint for extra strength and sealing effect, with bolts and rivets going through both sheet metal and angle iron.

Despite the lapping effects of the galvanized sheets and the additional pressence of the angle iron, the joints leaked.

The contractor tried making the joints tighter, but they still leaked. It was found that tiny cracks radiating from rivet holes caused the leaks. Reconstruction of the ducts using a heavier gauge metal would have eliminated possibility of the tiny cracks adjoining the rivet holes, but such an operation would have been costly and time consuming. The idea was to fill the leaks—and at the same time prevent possibility of further leaks.

After investigating a number of possible solutions, the answer was found in the Sun product, called Hornflex. A tough but flexible sealing compound which must be mixed with a hardener in the manner of epoxy, the Hornflex was applied to both inner and outer sides of the duct along the joints. The compound is described as effecting an almost molecular bond with any material it contacts. Insides of the joints were covered by having a man crawl through the ducts and apply the sealing compound with a spatula. Outsides of the joints were covered last. Corners were sealed with a caulking gun.

The same product was used recently —after other products had failed—to seal 5-in. expansion joints in the Algiers Lock at the Mississippi ship canal near New Orleans.

**Compact Nuclear Core.**—An experimental nuclear core designed for use in a compact reactor which would generate electricity directly without moving parts has been tested successfully, according to the Atomic Energy Commission. The core is only 7 in. high, 7 in. in diameter, one-fifth of a cubic foot in volume and weighs less than 200 lb.

The experimental core was designed and constructed for the Commission's Office for Aircraft Reactors by Atomics International, a division of North American Aviation, Inc., Canoga Park, Calif., as part of a Commission program to develop Systems for Nuclear Auxiliary Power (SNAP) for space vehicles. The experiment is located at a commission facility operated by the company in the Santa Susana Mountains 25 miles north of Los Angeles.

Initial nuclear operation (criticality) was achieved on August 2. The purpose of the experiment, which will be continued until November, is to obtain physics data for the development of a complete reactor system. No electricity is being generated.

The complete system, including a thermo-electric conversion device, is scheduled to be tested early next year. It has been designated SNAP 10.

The fuel and moderator in the experimental device are a homogeneous mixture of uranium 235 and zirconium hydride. The heat conduction and reflector units are made of beryllium. The device is divided into two identical units which were brought together by remote control to achieve a sustained chain reaction. Each half was built up with alternate layers of fuel-moderator disks and circular conduction plates within a reflector.

In the complete SNAP 10 system, thermocouples will surround the reactor core. Electricity will be generated by the flow of heat from the actor through the thermocouples. Excess heat, not used in the generation of the electricity, will flow through the thermocouples and then to radiator fins. The fins will radiate the heat to the atmosphere. The system, for which the core is designed, is expected to generate 300 watts of electricity.

**Radio Waves from Saturn Measured.**—University of Michigan scientists have measured radio waves from the planet Saturn. The University's head of radio astronomy, Professor Fred T. Haddock, reported these first clear and unambiguous detections and measurements of Saturn and a planetary nebula to the 13th General Assembly of the International Scientific Radio Union.

The findings are basic contributions to scientific knowledge. They could also help save money in future planetary exploration and possibly lead to modification of theories of planetary nebulae.

The measurements were made with the U-M's high precision 85-ft. radio telescope through the use of a new and highly-sensitive ruby maser amplifier. This amplifier was developed by the University's Willow Run Laboratories in an application of its earlier discovery, in 1957, of maser action in ruby.

Saturn's atmospheric temperature was found to be 100° K. (− 283° F.), about the temperature expected on the basis of optical studies. The naturally generated radio waves are emitted from various depths in the planet's atmosphere, Prof. Haddock pointed out.

The measurement may make possible study of the temperature and density of the rings of Saturn and of the distribution of gases in Saturn's atmosphere. Precise measurements at a number of frequencies are needed for this, Professor Haddock pointed out. The U-M findings are accurate within 10 per cent, which Haddock termed "pretty good."

**Two New Polymers.**—Two new functional polymers with a wide range of applications in cosmetic formulation, leather finishing, textile dyeing and stripping, tablet binding and coating, clarification and stabilization of beer and wine, and the formulation of waxes, polishes and inks have been developed by the Dow Chemical Company, Midland, Mich.

According to Dow, many organic complexes and several inorganic complexes can be prepared utilizing the new resins which will be marketed under the trademarks Devlex 130 and Devlex A515.

Complexing provides a means of modifying the solubility, volatility, stability, toxicity properties, and—in some cases—the odor and taste of complexed molecules without altering their chemical nature.

The film-forming properties of the Devlex resins are particularly adapted for use with such substances as cotton, wool, paper, skin, hair and leather.

Dow said the resins should find wide use in the cosmetic industry as fixatives in a variety of hair grooming preparations where it offers advantages in increased body sheen, natural feel, protection against excessive build-up of static electricity, and better control after shampooing.

Other actual or potential applications include tablet binding and coating, clarification and stabilization of beer and wine, dye leveling and stripping, leather finishing, and the formulation of waxes, polishes, and inks. Food and drug applications are being explored with special research grades of Devlex resins available now only in experimental quantities.

Further information and samples may be obtained from Dow's Technical Service and Development Department, Midland.

**Control of Air Contamination.**—Air in a part of General Electric's Power Tube factory is cleaner than the air in the arctic. The comparison is made

by the company's General Engineering Laboratory which has been conducting regular tests of the air in the "Snow White" room where minute parts of television camera pickup tubes (image orthicons) are assembled.

Detecting, counting and controlling invisible airborne particles so tiny that 50,000 of them laid end-to-end would equal only the thickness of a human hair, has become a necessary adjunct to TV camera-tube manufacture.

The company's physicists say the air in the ultra-clean factory room contains only about 3,000 of these invisible particles per cubic centimeter, while air in an air-conditioned office has 35,000 particles per cubic centimeter and the air in a heavy industrial city is loaded with 170,000 particles per cubic centimeter.

Control of air contamination in manufacturing and assembly areas is imperative. The thin-film target used in General Electric's "see-in-the-dark" tube (GL-7629), for instance, is only about two millionths of an inch thick. This is considerably thinner than most of the dust particles which can be seen dancing on a sunbeam.

A visible piece of dust on the target of an image orthicon, being used in a telecast, would appear as large as a boulder on the home viewer's screen.

Tests of the air cleanliness in the tube factory are made with a "condensation nuclei detector," developed by the General Engineering Laboratory of General Electric. This device can measure particles ranging in size from 0.001 micron (0.000000039 of an inch) to 0.1 micron (0.0000039 of an inch).

The "condensation nuclei detector" works on the principle that moisture condenses on each invisible airborne particle. With continuing condensation, the particles grow into visible droplets.

A light-scattering photometer which detects the forward scattering light from particles 0.2 to 50 microns is used to measure the larger particles.

The air is kept clean in the "Snow White" room by a combination of methods and devices which includes air-filtration and air-conditioning, frequent vacuum cleaning, air locks for entry and exit, use of lint-free clothing, hats and rubber finger cots.

**Students' Potentiometer.**—Leeds & Northrup Company's completely new 7645 Student's Potentiometer is designed for use by educators to demonstrate the potentiometer principle in measurements of voltage, current, temperature, hydrogen ion concentration (pH) and electrolytic conductivity. Despite its simplicity of design —making it an ideal teaching tool— it provides an accuracy suitable for use in research and development laboratories.

This ruggedly-built potentiometer has unusual design features never before available in an instrument of this type, including:

1. Central reading window, where the measured emf appears as a row of digits plus a scale interpolation for easier reading.

2. Three operating ranges, 0 to 1.6 volts, 0 to 0.16 volt and 0 to 0.016 volt to provide new versatility of operation.

3. High-quality selector switch assures low, stable contact resistances and low thermal emfs, and thereby more accurate measurements.

The limits of error are $\pm 0.0005$ volt on the high range, $\pm 0.0001$ volt on the medium range, and $\pm 0.00001$ volt on the low range. The potentiometer is housed in a gray metal case and is supplied with a vinyl dust cover.

**New Nebula Sighted.**—Radio-telescope investigations made by scientists at Cambridge, England, have led to the sighting of the new nebula by the telescope at Mount Palomar, Calif., at a greater distance in the universe than man has ever seen before. The Mullard Radio-Astronomy Observatory at Cambridge fixed the nebula's position at five thousand million light years away. From this information, the 200-in. telescope at the Mount Palomar Observatory successfully photographed the nebula.

The two observatories were in correspondence since signals were first noted on the Cambridge radio-telescopes last year.

The success of this joint enterprise is the result of regular cooperation between the observatories. Pin-point positions made by the Cambridge radio-telescopes have been used at Mount Palomar to find some of the most exciting objects in the universe—including entire galaxies in collision.

Radio signals from the nebula were recently relayed to Burlington House, London, where the tercentenary of the Royal Society—Britan's leading scientific institution—is currently being celebrated. Visitors were also able to hear a recording of sounds made from the intensive radio waves from the colliding galaxies in the Cygnus group.

These waves, according to Cambridge observatory scientists, set out toward Earth 500 million years ago—300 million years before the first dinosaurs appeared on Earth.

The observatory's radio-telescopes are among the largest in the world, although less spectacular than the versatile paraboloid-radio telescope at Jodrell Bank, England, which has been responsible for tracking America's deep space satellites.

**Pneumatic Shock Test Machine.**—A pneumatic powered shock test machine for production testing of small electronic components that produces up to 3000 g's has been announced by the Industrial Products Subdivision, Avco Research and Advanced Development Division, Wilmington, Mass. The machine handles specimen loads up to 30 lb. and produces all shock pulses required for testing electronic components with an impact shock velocity of up to 192 in. per second. This includes the 3000 g, 0.2 millisecond shock pulse required by MIL-S-19500B. The design of the new SM-005 represents a departure from customary free-fall shock machines in that it employs pneumatic actuation and control of the shock pulses produced.

The new machine has no superstructure and relies on control of air pressure into and out of a pneumatic cylinder. The specimen carriage is connected to the pneumatic cylinder by a guide rod and in operation is raised and propelled downward by a discharge of air.

Features of the new machine include one-hand carriage height control adjustment, automatic brake, re-cycling selector, pressure valve setting for g-levels desired and two-hand actuation requirement. As a safety factor this last feature insures that the operator must have both hands on the control panel at the time of a carriage impact.

Although the new machine is only 46 in. high over-all, when a drop height of 7 in. is employed at an air pressure setting of 80 lb., the machine develops an impact velocity of 192 in. per second. This is the equivalent of a 4-ft. free fall drop.

The Avco SM-005 Shock Machine is composed of 5 major assemblies: base, brake, carriage, control panel and actuator.

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